Note on the paper "An improvement of Batir's asymptotic formula and some estimates related to the gamma function"

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ABSTRACT. The aim of this note is to correct a result given in [Cristea, V. C. and Dumitrescu, S., *An improvement of Batir's asymptotic formula and some estimates related to the gamma function*, Creat. Math. Inform., **25** (2016), No. 2 165–173]

One of the purposes announced in [2] is to improve Batir's approximation formula [1]:

$$\Gamma(n+1) \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \frac{1}{3n} + \frac{1}{18n^2} - \frac{2}{405n^3} - \frac{31}{9720n^4}\right)^{\frac{1}{4}}, \text{ as } n \to \infty.$$
 (1)

The authors use in [2] a method to add new terms in (1), in order to obtain better results. Unfortunately, an inattention has appeared just in the beginning of the paper, as the authors of [2] incorrectly considered the term $-\frac{2}{425n^3}$ instead of $-\frac{2}{405n^3}$.

As a result, Theorem 2.1-2.2, Proposition 3.1 in [2] should not be considered, being not extensions of (1).

However, by using the method presented in [2], the following correct results can be stated:

$$\sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \frac{1}{3n} + \frac{1}{18n^2} - \frac{2}{405n^3} - \frac{31}{9720n^4}\right)^{\frac{1}{4} + \frac{529}{272160n^4}} < \Gamma\left(n+1\right),$$

for every integer $n \ge 19$ and

$$\Gamma\left(n+1\right) < \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \frac{1}{3n} + \frac{1}{18n^2} - \frac{2}{405n^3} - \frac{31}{9720n^4}\right)^{\frac{1}{4} + \frac{529}{272160n^4} + \frac{497}{3499200n^5}},$$

for every integer $n \ge 1$.

REFERENCES

[1] Batir, N., Very accurate approximations for the factorial function, J. Math. Inequal., 4 (2010), No. 3, 335-344

[2] Cristea, V. C. and Dumitrescu, S., An improvement of Batir's asymptotic formula and some estimates related to the gamma function, Creat. Math. Inform., 25 (2016), No. 2, 165–173

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Received: 01.12.2018. In revised form: 12.01.2019. Accepted: 12.03.2019 2010 Mathematics Subject Classification. 33B15, 26D15, 41A60). Key words and phrases. Batir's approximation formula, Gamma function.