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A comparative study of the PL homotopy and BFGS methods for some nonsmooth optimization problems

ANDREI BOZÂNTAN[1] and VASILE BERINDE[1,2]

ABSTRACT.

We consider some non-smooth functions and investigate the numerical behavior of the Piecewise Linear Hompotopy (PLH) method ([Bozântan, A., *An implementation of the piecewise-linear homotopy algorithm for the computation of fixed points*, Creat. Math. Inform., **19** (2010), No. 2, 140–148] and [Bozântan, A. and Berinde, V., *Applications of the PL homotopy algorithm for the computation of fixed points to unconstrained optimization problems*, Creat. Math. Inform., **22** (2013), No. 1, 41–46]). We compare the PLH method with the BFGS with inexact line search, a quasi-Newton method, having some results reported in [Lewis, A. S. and Overton, M. L., *Nonsmooth optimization via BFGS*, submitted to SIAM J. Optimiz, (2009)]. For most of the considered cases, the characteristics of the PLH method are quite similar to the BFGS method, that is, the PLH method converges to local minimum values and the convergence rate seems to be linear with respect to the number of function evaluations, but we also identify some issues with the PLH method.

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[1] DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
NORTH UNIVERSITY CENTRE AT BAIA MARE
VICTORIEI 76, 430122 BAIA MARE, ROMANIA
E-mail address: andrei.bozantan@gmail.com
E-mail address: vberinde@cunbm.utcluj.ro

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[2] ACADEMY OF ROMANIAN SCIENTISTS
E-mail address: vberinde@cunbm.utcluj.ro