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D-Fuzzy set and its applications in decision making problems

JULEE SRIVASTAVA and SUDHIR MADDHESHIYA

ABSTRACT. In this paper we have presented an approach to deal with decision making problems. For this purpose we have constructed a fuzzy set, namely D-fuzzy set, for a soft set. This concept of D-fuzzy set will also be used when the information is given in form of weighted soft set, fuzzy soft set or weighted fuzzy soft set. Further we will apply this method when there are more than one observer. We will define such sets as multi-observer soft set, multi-observer weighted soft set etc.

1. INTRODUCTION

Often we have to deal with problems that involve various types of uncertainties. These types of problems cannot be dealt with the available classical tools. There are many theories developed for dealing with them. Some of them are probability theory, theory of fuzzy sets [22], theory of intuitionistic fuzzy sets [2, 3], theory of vague sets [9], theory of interval mathematics [17] and theory of rough sets [19]. All of these theories have their own advantages and some limitations as well. One major drawback of these theories is probably the inadequacy of parameterization tools which was observed by Molodtsov in 1999. Consequently he introduced the concept of soft set theory [16] that is free from the difficulties that have troubled the usual theoretical approaches. The soft set theory involves parameterization technique. Molodtsov provided several applications of soft set theory in his work.

Maji et al. [15] further developed the basic theory of soft sets. Using attributes reduction in rough set theory, Maji et al. [14] presented an application of soft set in decision making problem to reduce parameter set of a soft set. But Chen [6] pointed out that unreal optimal choice objects may be also obtained through this way and showed that the method of attributes reduction in rough set theory can not simply transplant to parameters reduction in soft set theory. Roy et al. [21] presented a comparison score based approach dealing with the fuzzy soft set based decision making problems. In this approach, they compare the membership values of two objects with respect to a common attribute to determine which one relatively possesses that attribute. Kong et al. [10] revised this method and their revision (the fuzzy choice value based method) has been proved as another method based on the maximum fuzzy choice value. Feng et al. [8] presented a novel approach to fuzzy soft set based decision making problems by using level soft sets. They investigated the fuzzy soft set based decision making problems more deeply and their new method can be successfully applied to some decision making problems. Razak et al. [20] used the method of lambda – max to determine the weight of parameters and presented an algorithm for solving decision making problems. Nagarajan et al. [18]

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Corresponding author: Sudhir Maddheshiya; sudhirmaddy149@gmail.com

investigated fuzzy soft matrix, their operations and fuzzy soft algorithm that allows constructing those efficient decision making method. Alhazaymeh et al. [1] gave applications of generalized vague soft expert set in decision making. Lalotra et al. [11] introduced an HF-knowledge measure and showed its effectiveness with the help of an illustrative example from the viewpoint of linguistic hedges and applied the proposed HF-knowledge measure to multiple-attribute decision-making (MADM) problem by utilizing the TOPSIS method and justified its advantage over existing HF-entropies. From [4], [5] and [7] we got idea to construct a fuzzy set which will help us in decision making.

In this paper we are going to construct a fuzzy set with the help of choice values. This fuzzy set will be called D-fuzzy set (or decision making fuzzy set) for a given soft set. This D-fuzzy set will give us the optimal object on the basis of the parameters in decision making problem. We will use this method even in the case of weighted soft sets, fuzzy soft sets and weighted fuzzy soft sets further in this paper.

Usually there is only one observer in a soft who observe the universe objects. But this creates biasedness. So we will also treat the cases when there are more than one observer and we will call them multi-observer soft sets, multi-observer weighted soft sets etc. We will also see through examples that using multi-observer soft set, we will get more precise results.

Our paper is organized as follows: In section **1** we have given some intuitive introduction. In section **2** we have given some basic definitions. In section **3** we have constructed D-fuzzy set for soft sets with only one observer and given applications of this through examples. In section **4** we have constructed D-fuzzy set for soft sets with more than one observers and given applications of this through examples. In section **5** we give the conclusion and indicate possible further developments.

2. Preliminaries

Let U be a universe set, E be a set of parameters, X be a set of observers and W be set of weights (a positive real number less than or equal to 1) imposed on the parameters of the set E.

Definition 2.1 (Fuzzy set)**.** [22] A pair $S = (U, \mu)$ is said to be a fuzzy set over U, where $\mu : U \longrightarrow [0, 1]$ is called the membership function of A.

Definition 2.2 (Soft set)**.** [16] A pair $S = (F, E)$ is said to be a soft set over U, where $F : E \longrightarrow P(U)$ is a mapping from E to $P(U)$ (set of all crisp subsets of U).

Definition 2.3 (Weighted soft set)**.** [12] A pair $S = (F, E|W)$ is said to be a weighted soft set over U, where

$$E|W = \{e|w : e \in E, w \in W \text{ is weight imposed on } e\}$$

and $F : E|W \longrightarrow P(U)$ is a mapping from $E|W$ to $P(U)$.

Definition 2.4 (Fuzzy soft set)**.** [13] A pair $S = (F, E)$ is said to be a fuzzy soft set over U, where $F : E \longrightarrow \mathcal{F}(U)$ is a mapping from E to $\mathcal{F}(U)$ (set of all fuzzy sets over U).

Definition 2.5 (Weighted fuzzy soft set)**.** [8] A pair $S = (F, E|W)$ is said to be a weighted fuzzy soft set over U, where $E|W$ is as defined in Definition 2.3 and $F : E|W \longrightarrow \mathcal{F}(U)$ is a mapping from $E|W$ to $\mathcal{F}(U)$.

Remark 2.1. In the definitions 2.2, 2.3, 2.4, 2.5 given above, we have considered only one observer and F describes opinion of that observer. But we often have to include more than one observer to get the more precise decisions.

For more than one observer we will use the following definitions.

Definition 2.6 (Multi-observer soft set)**.** A pair $S = (F, A)$ is said to be a multi-observer soft set over U, where $A = E \times X$ and $F : A \longrightarrow P(U)$ is a mapping from A to $P(U)$ (set of all crisp subsets of U).

Definition 2.7 (Multi-observer weighted soft set)**.** A pair $S = (F, A|W)$ is said to be a multi-observer weighted soft set over U, where $A|W = E|W \times X$ is defined as

$$A|W = \{(e|w, x) : e \in E, w \in W \text{ is weight imposed on } e \text{ and } x \in X\}$$

and $F : A|W \longrightarrow P(U)$ is a mapping from $A|W$ to $P(U)$.

Definition 2.8 (Multi-observer fuzzy soft set)**.** A pair $S = (F, A)$ is said to be a multi-observer fuzzy soft set over U, where $A = E \times X$ and $F : A \longrightarrow \mathcal{F}(U)$ is a mapping from A to $\mathcal{F}(U)$ (set of all fuzzy sets over U).

Definition 2.9 (Multi-observer weighted fuzzy soft set)**.** A pair $S = (F, A|W)$ is said to be a multi-observer weighted fuzzy soft set over U, where $A|W = E|W \times X$ is as defined in Definition 2.7 and $F : A|W \longrightarrow \mathcal{F}(U)$ is a mapping from $A|W$ to $\mathcal{F}(U)$.

3. Construction of D-fuzzy set

If there is only one observer in the soft set then we will add up all the values of the characteristic function (when fuzziness is not included in soft set) or the membership function (when fuzziness is included) of $F(e_j)$ for all parameters e_j corresponding to an object. If there are weights associated to the parameters then we will multiply the values of parameters by the corresponding weights.

Dividing this sum by cardinality of set of parameters, we obtain D-fuzzy set for given kind of soft set.

3.1. **Construction of D-fuzzy set for a soft set.**

Suppose we are given a soft set (F, E) over the universe $U = \{u_1, u_2, \ldots\ldots, u_m\}$, where $E = \{e_1, e_2, \ldots\ldots, e_n\}$ is set of parameters and $F : E \longrightarrow P(U)$ is a mapping. For this soft set we construct D-fuzzy set $D = (U, \mu)$ over U, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E|} \sum_{j=1}^{n} \chi_{F(e_j)}(u_i)$$

where $\chi_{F(e_j)} : U \longrightarrow \{0, 1\}$ is the characteristic function of $F(e_j)$, defined by

$$\chi_{F(e_j)}(u_i) = \begin{cases} 1 & if\ u_i \in F(e_j) \\ 0 & if\ u_i \notin F(e_j) \end{cases}$$

3.2. **Construction of D-fuzzy set for a weighted soft set.**

Suppose we are given a weighted soft set $(F, E|W)$ over the universe $U = \{u_1, u_2, \ldots\ldots\ldots, u_m\}$, where $E = \{e_1, e_2, \ldots\ldots\ldots, e_n\}$ is set of parameters and $F : E|W \longrightarrow P(U)$ is a mapping. Here $W = \{w_1, w_2, \ldots\ldots\ldots, w_n\}$ is set of weights imposed on the parameters of the set E in such a way that w_i is the weight imposed on e_i for every $1 \leq i \leq n$. For this weighted soft set we construct D-fuzzy set $D = (U, \mu)$, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E|} \sum_{j=1}^{n} \chi_{F(e_j|w_j)}(u_i).w_j$$

where $\chi_{F(e_j|w_j)} : U \longrightarrow \{0, 1\}$ is the characteristic function of $F(e_j|w_j)$ defined by

$$\chi_{F(e_j|w_j)}(u_i) = \begin{cases} 1 & if\ u_i \in F(e_j|w_j) \\ 0 & if\ u_i \notin F(e_j|w_j) \end{cases}$$

3.3. **Construction of D-fuzzy set for a fuzzy soft set.** Suppose we are given a fuzzy soft set (F, E) over the universe $U = \{u_1, u_2, \dots\dots, u_m\}$, where $E = \{e_1, e_2, \dots\dots, e_n\}$ is set of parameters and $F : E \longrightarrow \mathcal{F}(U)$ is a mapping. For this fuzzy soft set we construct D-fuzzy set $D = (U, \mu)$ over U, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E|} \sum_{j=1}^{n} \mu_{F(e_j)}(u_i)$$

where $\mu_{F(e_j)} : U \longrightarrow [0, 1]$ is the membership function of the fuzzy set $F(e_j)$.

3.4. **Construction of D-fuzzy set for a weighted fuzzy soft set.** Suppose we are given a weighted fuzzy soft set $(F, E|W)$ over the universe $U = \{u_1, u_2, \dots\dots\dots, u_m\}$, where $E = \{e_1, e_2, \dots\dots\dots, e_n\}$ is set of parameters and $F : E|W \longrightarrow \mathcal{F}(U)$ is a mapping. Here $W = \{w_1, w_2, \dots\dots\dots, w_n\}$ is set of weights imposed on the parameters of the set E in such a way that w_i is the weight imposed on e_i for every $1 \leq i \leq n$. For this weighted soft set we construct D-fuzzy set $D = (U, \mu)$, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E|} \sum_{j=1}^{n} \mu_{F(e_j|w_j)}(u_i).w_j$$

where $\mu_{F(e_j|w_j)} : U \longrightarrow [0, 1]$ is the membership function of $F(e_j|w_j)$.

Note: After constructing D-fuzzy set we find m_0 such that $\mu(u_{m_0}) = \max\limits_{1 \leq i \leq m} \mu(u_i)$. Then m_0 is the optimal choice object. If m_0 has more than one value then any one of them could be chosen.

3.5. **Application of D-fuzzy set in decision making problem when the given information is in form of a soft set.** Suppose $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a collection of houses (universe set) and Mr. X has to buy one of these houses on the basis of the following set of parameters : $E = \{e_1 = beautiful, e_2 = modern, e_3 = in\ green\ surroundings, e_4 = cheap, e_5 = in\ good\ repair\}$.

The corresponding soft set is given by (F, E), where

$$\begin{aligned} F(e_1) &= \{h_1, h_3, h_5, h_6\} \\ F(e_2) &= \{h_2, h_4, h_6\} \\ F(e_3) &= \{h_1, h_2, h_3, h_6\} \\ F(e_4) &= \{h_3, h_5\} \\ F(e_5) &= \{h_1, h_5, h_6\}) \end{aligned}$$

Using 3.1, D-fuzzy set for this soft set, is given by (U, μ), where μ is defined by

$$\mu(h_i) = \frac{1}{|E|} \sum_{j=1}^{5} \chi_{F(e_j)}(h_i)$$

Hence,

$$\begin{aligned} \mu(h_1) &= \frac{1}{5}(1 + 0 + 1 + 0 + 1) = \frac{3}{5} \\ \mu(h_2) &= \frac{1}{5}(0 + 1 + 1 + 0 + 0) = \frac{2}{5} \\ \mu(h_3) &= \frac{1}{5}(1 + 0 + 1 + 1 + 0) = \frac{3}{5} \\ \mu(h_4) &= \frac{1}{5}(0 + 1 + 0 + 0 + 0) = \frac{1}{5} \end{aligned}$$

$$\mu(h_5) = \frac{1}{5}(1+0+0+1+1) = \frac{3}{5}$$
$$\mu(h_6) = \frac{1}{5}(1+1+1+0+1) = \frac{4}{5}$$

Since $\mu(h_6) = \max_{1\leq i\leq 6} \mu(h_i)$, therefore Mr. X should buy the house h_6.

3.6. **Application of D-fuzzy set in decision making problem when the given information is in form of a weighted soft set.** Let $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ be a set of candidates appearing for an interview, from which one has to be employed for a job in a company on the basis of the following set of parameters : $E = \{e_1 = good\ communication\ skill$, $e_2 = good\ knowledge\ in\ the\ required\ field, e_3 = minimum\ five\ years\ experience, e_4 = good\ previous\ record\}$. Let $w_1 = 0.8, w_2 = 0.9, w_3 = 0.7, w_4 = 0.85$ be the weights imposed on the parameters e_1, e_2, e_3, e_4 respectively.

The corresponding weighted soft set is given by $(F, E|W)$, where

$$F(e_1|w_1) = \{c_1, c_3, c_5, c_6\}$$
$$F(e_2|w_2) = \{c_1, c_2, c_4, c_5, c_7\}$$
$$F(e_3|w_3) = \{c_1, c_3, c_4, c_6\}$$
$$F(e_4|w_4) = \{c_2, c_3, c_4, c_5, c_7\}$$

Using 3.2, D-fuzzy set for this weighted soft set, is given by (U, μ) where μ is defined by

$$\mu(c_i) = \frac{1}{|E|}\sum_{j=1}^{4} \chi_{F(e_j|w_j)}(c_i)$$

Hence,

$$\mu(c_1) = \frac{1}{4}(0.8+0.9+0.7+0) = 0.6$$
$$\mu(c_2) = \frac{1}{4}(0+0.9+0+0.85) = 0.4375$$
$$\mu(c_3) = \frac{1}{4}(0.8+0+0.7+0.85) = 0.5875$$
$$\mu(c_4) = \frac{1}{4}(0+0.9+0.7+0.85) = 0.6125$$
$$\mu(c_5) = \frac{1}{4}(0.8+0.9+0+0.85) = 0.6375$$
$$\mu(c_6) = \frac{1}{4}(0.8+0+0.7+0) = 0.375$$
$$\mu(c_7) = \frac{1}{4}(0+0.9+0+0.85) = 0.4375$$

Since $\mu(c_5) = \max_{1\leq i\leq 5} \mu(c_i)$, therefore candidate c_5 should be employed.

3.7. **Application of D-fuzzy set in decision making problem when the given information is in form of a fuzzy soft set.** Let $U = \{m_1, m_2, m_3, m_4, m_5, m_6\}$ be a set of six mobiles and Mr. Y has to purchase one of them on the basis of the following set of parameters: $E = \{e_1 = good\ display\ resolution, e_2 = good\ camera\ quality, e_3 = good\ software\ features, e_4 = long\ battery\ life, e_5 = high\ storage\}$.

The corresponding fuzzy soft set is given by (F, E), where

$$F(e_1) = \left\{\frac{0.7}{m_1}, \frac{0.75}{m_2}, \frac{0.8}{m_3}, \frac{0.9}{m_4}, \frac{0.7}{m_5}, \frac{0.85}{m_6}\right\}$$

$$F(e_2) = \left\{ \frac{0.65}{m_1}, \frac{0.6}{m_2}, \frac{0.8}{m_3}, \frac{0.8}{m_4}, \frac{0.8}{m_5}, \frac{0.85}{m_6} \right\}$$
$$F(e_3) = \left\{ \frac{0.75}{m_1}, \frac{0.8}{m_2}, \frac{0.85}{m_3}, \frac{0.9}{m_4}, \frac{0.95}{m_5}, \frac{0.75}{m_6} \right\}$$
$$F(e_4) = \left\{ \frac{0.8}{m_1}, \frac{0.65}{m_2}, \frac{0.8}{m_3}, \frac{0.85}{m_4}, \frac{0.9}{m_5}, \frac{0.7}{m_6} \right\}$$
$$F(e_5) = \left\{ \frac{0.85}{m_1}, \frac{0.85}{m_2}, \frac{0.75}{m_3}, \frac{0.95}{m_4}, \frac{0.85}{m_5}, \frac{0.7}{m_6} \right\}$$

Using 3.3, D-fuzzy set for this fuzzy soft set, is given by (U, μ) where μ is defined by

$$\mu(m_i) = \frac{1}{|E|} \sum_{j=1}^{5} \mu_{F(e_j)}(m_i)$$

Hence,

$$\mu(m_1) = \frac{1}{5}(0.7 + 0.65 + 0.75 + 0.8 + 0.85) = 0.75$$
$$\mu(m_2) = \frac{1}{5}(0.75 + 0.6 + 0.8 + 0.65 + 0.85) = 0.73$$
$$\mu(m_3) = \frac{1}{5}(0.8 + 0.8 + 0.85 + 0.8 + 0.75) = 0.8$$
$$\mu(m_4) = \frac{1}{5}(0.9 + 0.8 + 0.9 + 0.85 + 0.95) = 0.88$$
$$\mu(m_5) = \frac{1}{5}(0.7 + 0.8 + 0.95 + 0.9 + 0.85) = 0.84$$
$$\mu(m_6) = \frac{1}{5}(0.85 + 0.85 + 0.75 + 0.7 + 0.7) = 0.77$$

Since $\mu(m_4) = \max\limits_{1 \le i \le 6} \mu(m_i)$, therefore Mr. Y should purchase the mobile m_4.

3.8. **Application of D-fuzzy set in decision making problem when given information is in form of a weighted fuzzy soft set.** Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ be a collection of cars and Mr. Z wants to buy one of them on the basis of the following set of parameters : $E = \{e_1 = fuel\ efficiency, e_2 = beautiful, e_3 = spacious, e_4 = eco\ friendly, e_5 = high\ security\ measure\}$. Let $w_1 = 0.95, w_2 = 0.8, w_3 = 0.85, w_4 = 0.8, w_5 = 0.9$ be the weights imposed on the parameters e_1, e_2, e_3, e_4, e_5 respectively.

The corresponding weighted fuzzy soft set is given by $(F, E|W)$, where

$$F(e_1|w_1) = \left\{ \frac{0.8}{c_1}, \frac{0.9}{c_2}, \frac{0.85}{c_3}, \frac{0.75}{c_4}, \frac{0.85}{c_5}, \right\}$$
$$F(e_2|w_2) = \left\{ \frac{0.85}{c_1}, \frac{0.95}{c_2}, \frac{0.8}{c_3}, \frac{0.85}{c_4}, \frac{0.85}{c_5} \right\}$$
$$F(e_3|w_3) = \left\{ \frac{0.8}{c_1}, \frac{0.8}{c_2}, \frac{0.85}{c_3}, \frac{0.95}{c_4}, \frac{0.8}{c_5} \right\}$$
$$F(e_4|w_4) = \left\{ \frac{0.65}{c_1}, \frac{0.8}{c_2}, \frac{0.85}{c_3}, \frac{0.9}{c_4}, \frac{0.75}{c_5} \right\}$$
$$F(e_5|w_5) = \left\{ \frac{0.75}{c_1}, \frac{0.85}{c_2}, \frac{0.75}{c_3}, \frac{0.85}{c_4}, \frac{0.75}{c_5} \right\}$$

Using 3.4, D-fuzzy set for this weighted fuzzy soft set, is given by (U, μ), where μ is defined by

$$\mu(c_i) = \frac{1}{|E|} \sum_{j=1}^{5} \mu_{F(e_j|w_j)}(c_i).w_j$$

Hence,

$$\begin{aligned}
\mu(c_1) &= \frac{1}{5}(0.8 \times 0.95 + 0.85 \times 0.8 + 0.8 \times 0.85 + 0.65 \times 0.8 + 0.75 \times 0.9) \\
&= 0.663 \\
\mu(c_2) &= \frac{1}{5}(0.9 \times 0.95 + 0.95 \times 0.8 + 0.8 \times 0.85 + 0.8 \times 0.8 + 0.85 \times 0.9) \\
&= 0.74 \\
\mu(c_3) &= \frac{1}{5}(0.85 \times 0.95 + 0.8 \times 0.8 + 0.85 \times 0.85 + 0.85 \times 0.8 + 0.75 \times 0.9) \\
&= 0.705 \\
\mu(c_4) &= \frac{1}{5}(0.75 \times 0.95 + 0.85 \times 0.8 + 0.95 \times 0.85 + 0.9 \times 0.8 + 0.85 \times 0.9) \\
&= 0.737 \\
\mu(c_5) &= \frac{1}{5}(0.85 \times 0.95 + 0.85 \times 0.8 + 0.8 \times 0.85 + 0.75 \times 0.8 + 0.75 \times 0.9) \\
&= 0.6885
\end{aligned}$$

Since $\mu(c_2) = \max_{1 \leq i \leq 5} \mu(c_i)$, therefore Mr. Z should buy the car c_2.

4. CONSTRUCTION OF D-FUZZY SET IN CASE OF MULTI-OBSERVER

In the case of more than one observer the value of a parameter will be replaced by the average of the values of characteristic function or the membership function for all observers. Including more than one observers in our problem we will get more satisfactory results.

Dividing this sum by cardinality of set of parameters, we obtain D-fuzzy set for given kind of soft set.

4.1. **Construction of D-fuzzy set for a multi-observer soft set.** Suppose we are given a multi-observer soft set (F, A) over the universe $U = \{u_1, u_2, \ldots\ldots, u_m\}$, where $A = E \times X$, $E = \{e_1, e_2, \ldots\ldots, e_n\}$ is set of parameters, $X = \{x_1, x_2, \ldots\ldots, x_p\}$ is set of observers and $F : A \longrightarrow P(U)$ is a mapping. For this multi-observer soft set we construct D-fuzzy set $D = (U, \mu)$ over U, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E||X|} \sum_{j=1}^{n} \sum_{k=1}^{p} \chi_{F(e_j,x_k)}(u_i)$$

where $\chi_{F(e_j,x_k)} : U \longrightarrow \{0, 1\}$ is the characteristic function of $F(e_j, x_k)$, defined by

$$\chi_{F(e_j,x_k)}(u_i) = \begin{cases} 1 & if\ u_i \in F(e_j, x_k) \\ 0 & if\ u_i \notin F(e_j, x_k) \end{cases}$$

4.2. **Construction of D-fuzzy set for a multi-observer weighted soft set.** Suppose we are given a multi-observer weighted soft set $(F, A|W)$ over the universe $U = \{u_1, u_2, \ldots\ldots\ldots, u_m\}$, where $A|W = E|W \times X$, $E = \{e_1, e_2, \ldots\ldots\ldots, e_n\}$ is set of parameters, $X = \{x_1, x_2, \ldots\ldots, x_p\}$ is set of observers and $F : A|W \longrightarrow P(U)$ is a mapping. Here $W = \{w_1, w_2, \ldots\ldots\ldots, w_n\}$ is set of weights imposed on the parameters of the set E in such a way that w_i is the weight imposed on e_i for every $1 \leq i \leq n$. For this multi-observer weighted soft set we construct D-fuzzy set $D = (U, \mu)$, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E||X|} \sum_{j=1}^{n} \left(\sum_{k=1}^{p} \chi_{F(e_j|w_j, x_k)}(u_i) \right) .w_j$$

where $\chi_{F(e_j|w_j,x_k)} : U \longrightarrow \{0, 1\}$ is the characteristic function of $F(e_j|w_j, x_k)$ defined by

$$\chi_{F(e_j|w_j,x_k)}(u_i) = \begin{cases} 1 & if\ u_i \in F(e_j|w_j, x_k) \\ 0 & if\ u_i \notin F(e_j|w_j, x_k) \end{cases}$$

4.3. **Construction of D-fuzzy set for a multi-observer fuzzy soft set.** Suppose we are given a multi-observer fuzzy soft set (F, A) over the universe $U = \{u_1, u_2, \ldots\ldots, u_m\}$, where $A = E \times X$, $E = \{e_1, e_2, \ldots\ldots, e_n\}$ is set of parameters, $X = \{x_1, x_2, \ldots\ldots, x_p\}$ is set of observers and $F : A \longrightarrow \mathcal{F}(U)$ is a mapping. For this multi-observer fuzzy soft set we construct D-fuzzy set $D = (U, \mu)$ over U, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E||X|} \sum_{j=1}^{n} \sum_{k=1}^{p} \mu_{F(e_j, x_k)}(u_i)$$

where $\mu_{F(e_j,x_k)} : U \longrightarrow [0, 1]$ is the membership function of the fuzzy set $F(e_j, x_k)$.

4.4. **Construction of D-fuzzy set for a multi-observer weighted fuzzy soft set.** Suppose we are given a multi-observer weighted fuzzy soft set $(F, A|W)$ over the universe $U = \{u_1, u_2, \ldots\ldots\ldots, u_m\}$, where $A|W = E|W \times X$, $E = \{e_1, e_2, \ldots\ldots\ldots, e_n\}$ is set of parameters $X = \{x_1, x_2, \ldots\ldots, x_p\}$ is set of observers and $F : A|W \longrightarrow \mathcal{F}(U)$ is a mapping. Here $W = \{w_1, w_2, \ldots\ldots\ldots, w_n\}$ is set of weights imposed on the parameters of the set E in such a way that w_i is the weight imposed on e_i for every $1 \leq i \leq n$. For this weighted soft set we construct D-fuzzy set $D = (U, \mu)$, whose membership function μ is given by

$$\mu(u_i) = \frac{1}{|E||X|} \sum_{j=1}^{n} \left(\sum_{k=1}^{p} \mu_{F(e_j|w_j, x_k)}(u_i) \right) .w_j$$

where $\mu_{F(e_j|w_j,x_k)} : U \longrightarrow [0, 1]$ is the membership function of $F(e_j|w_j, x_k)$.

Note: After constructing D-fuzzy set we find m_0 such that $\mu(u_{m_0}) = \max_{1 \leq i \leq m} \mu(u_i)$. Then m_0 is the optimal choice object. If m_0 has more than one value then any one of them could be chosen.

4.5. **Application of D-fuzzy set in decision making problem when the given information is in form of a multi-observer soft set.** Suppose there are three observers $x_1, x_2, x_3 \in X$ in example 3.5 who observe the universe objects.

The corresponding multi-observer soft set is given by (F, A), where

$$F(e_1, x_1) = \{h_1, h_3, h_4, h_5, h_6\}$$
$$F(e_1, x_2) = \{h_1, h_2, h_3, h_4, h_5\}$$
$$F(e_1, x_3) = \{h_3, h_4, h_5, h_6\}$$

$$\begin{aligned}
F(e_2, x_1) &= \{h_1, h_5, h_6\}\\
F(e_2, x_2) &= \{h_1, h_3, h_6\}\\
F(e_2, x_3) &= \{h_1, h_3, h_4, h_6\}\\
F(e_3, x_1) &= \{h_1, h_3, h_5, h_6\}\\
F(e_3, x_2) &= \{h_2, h_5, h_6\}\\
F(e_3, x_3) &= \{h_2, h_4, h_5, h_6\}\\
F(e_4, x_1) &= \{h_1, h_2, h_3, h_6\}\\
F(e_4, x_2) &= \{h_3, h_4, h_5\}\\
F(e_4, x_3) &= \{h_2, h_5, h_6\}\\
F(e_5, x_1) &= \{h_2, h_3, h_4, h_6\}\\
F(e_5, x_2) &= \{h_1, h_2, h_3, h_5, h_6\}\\
F(e_5, x_3) &= \{h_1, h_4, h_5\}
\end{aligned}$$

Using 4.1, D-fuzzy set for this multi-observer soft set, is given by (U, μ), where μ is defined by

$$\mu(h_i) = \frac{1}{|E||X|} \sum_{j=1}^{5} \sum_{k=1}^{3} \chi_{F(e_j, x_k)}(h_i)$$

Hence,

$$\begin{aligned}
\mu(h_1) &= \frac{1}{5 \cdot 3}(2 + 3 + 1 + 1 + 2) = \frac{9}{15}\\
\mu(h_2) &= \frac{1}{5 \cdot 3}(1 + 0 + 2 + 2 + 2) = \frac{7}{15}\\
\mu(h_3) &= \frac{1}{5 \cdot 3}(3 + 2 + 1 + 2 + 2) = \frac{10}{15}\\
\mu(h_4) &= \frac{1}{5 \cdot 3}(3 + 1 + 1 + 1 + 2) = \frac{8}{15}\\
\mu(h_5) &= \frac{1}{5 \cdot 3}(3 + 1 + 3 + 2 + 2) = \frac{11}{15}\\
\mu(h_6) &= \frac{1}{5 \cdot 3}(2 + 3 + 3 + 2 + 2) = \frac{12}{15}
\end{aligned}$$

Since $\mu(h_6) = \max_{1 \le i \le 6} \mu(h_i)$, therefore Mr. X should buy the house h_6.

4.6. **Application of D-fuzzy set in decision making problem when the given information is in form of a multi-observer weighted soft set.** Suppose there are four observers $x_1, x_2, x_3, x_4 \in X$ in example 3.6 who observe the universe objects.

The corresponding multi-observer weighted soft set is given by $(F, A|W)$, where

$$\begin{aligned}
F(e_1|w_1, x_1) &= \{c_1, c_2, c_3, c_4\}\\
F(e_1|w_1, x_2) &= \{c_1, c_3, c_5, c_7\}\\
F(e_1|w_1, x_3) &= \{c_1, c_2, c_3, c_4, c_6, c_7\}\\
F(e_1|w_1, x_4) &= \{c_1, c_3, c_4, c_5\}\\
F(e_2|w_2, x_1) &= \{c_1, c_2, c_3, c_5, c_7\}\\
F(e_2|w_2, x_2) &= \{c_1, c_2, c_3, c_5, c_7\}\\
F(e_2|w_2, x_3) &= \{c_1, c_3, c_5\}
\end{aligned}$$

$$F(e_2|w_2, x_4) = \{c_2, c_3, c_4, c_5, c_6\}$$
$$F(e_3|w_3, x_1) = \{c_2, c_3, c_4, c_5\}$$
$$F(e_3|w_3, x_2) = \{c_4, c_5, c_7\}$$
$$F(e_3|w_3, x_3) = \{c_2, c_3, c_5, c_6, c_7\}$$
$$F(e_3|w_3, x_4) = \{c_1, c_2, c_3, c_4, c_5, c_7\}$$
$$F(e_4|w_4, x_1) = \{c_1, c_3, c_4, c_5\}$$
$$F(e_4|w_4, x_2) = \{c_1, c_4, c_6, c_7\}$$
$$F(e_4|w_4, x_3) = \{c_2, c_3, c_4, c_5, c_6\}$$
$$F(e_4|w_4, x_4) = \{c_2, c_3, c_4, c_5, c_7\}$$

Using 4.2, D-fuzzy set for this multi-observer weighted soft set, is given by (U, μ) where μ is defined by

$$\mu(c_i) = \frac{1}{|E||X|} \sum_{j=1}^{4} \left(\sum_{k=1}^{4} \chi_{F(e_j|w_j, x_k)}(c_i) \right) .w_j$$

Hence,

$$\mu(c_1) = \frac{1}{4 \cdot 4}(4w_1 + 3w_2 + w_3 + 2w_4) = \frac{1}{16}(8.3) = 0.51875$$
$$\mu(c_2) = \frac{1}{4 \cdot 4}(2w_1 + 3w_2 + 3w_3 + 2w_4) = \frac{1}{16}(8.1) = 0.50625$$
$$\mu(c_3) = \frac{1}{4 \cdot 4}(4w_1 + 4w_2 + 3w_3 + 3w_4) = \frac{1}{16}(11.45) = 0.715625$$
$$\mu(c_4) = \frac{1}{4 \cdot 4}(3w_1 + w_2 + 3w_3 + 4w_4) = \frac{1}{16}(8.8) = 0.55$$
$$\mu(c_5) = \frac{1}{4 \cdot 4}(2w_1 + 4w_2 + 4w_3 + 3w_4) = \frac{1}{16}(10.55) = 0.659375$$
$$\mu(c_6) = \frac{1}{4 \cdot 4}(w_1 + w_2 + w_3 + 2w_4) = \frac{1}{16}(4.1) = 0.25625$$
$$\mu(c_7) = \frac{1}{4 \cdot 4}(2w_1 + 2w_2 + 3w_3 + 2w_4) = \frac{1}{16}(7.2) = 0.45$$

Since $\mu(c_3) = \max\limits_{1 \le i \le 5} \mu(c_i)$, therefore candidate c_3 should be employed.

Here one can see that including more than observer we have got c_3 as optimal candidate as compared to 3.6 where we had got c_5 as optimal candidate.

4.7. **Application of D-fuzzy set in decision making problem when the given information is in form of a multi-observer fuzzy soft set.** Suppose there are three observers $x_1, x_2, x_3 \in X$ in example 3.7 who observe the universe objects.

The corresponding multi-observer fuzzy soft set is given by (F, A), where

$$F(e_1, x_1) = \left\{ \frac{0.7}{m_1}, \frac{0.75}{m_2}, \frac{0.8}{m_3}, \frac{0.9}{m_4}, \frac{0.7}{m_5}, \frac{0.85}{m_6} \right\}$$
$$F(e_1, x_2) = \left\{ \frac{0.6}{m_1}, \frac{0.7}{m_2}, \frac{0.85}{m_3}, \frac{0.95}{m_4}, \frac{0.75}{m_5}, \frac{0.8}{m_6} \right\}$$
$$F(e_1, x_3) = \left\{ \frac{0.5}{m_1}, \frac{0.65}{m_2}, \frac{0.75}{m_3}, \frac{0.9}{m_4}, \frac{0.65}{m_5}, \frac{0.9}{m_6} \right\}$$
$$F(e_2, x_1) = \left\{ \frac{0.7}{m_1}, \frac{0.8}{m_2}, \frac{0.85}{m_3}, \frac{0.85}{m_4}, \frac{0.7}{m_5}, \frac{0.85}{m_6} \right\}$$

$$F(e_2, x_2) = \left\{ \frac{0.85}{m_1}, \frac{0.7}{m_2}, \frac{0.8}{m_3}, \frac{0.85}{m_4}, \frac{0.6}{m_5}, \frac{0.75}{m_6} \right\}$$
$$F(e_2, x_3) = \left\{ \frac{0.75}{m_1}, \frac{0.65}{m_2}, \frac{0.8}{m_3}, \frac{0.85}{m_4}, \frac{0.6}{m_5}, \frac{0.9}{m_6} \right\}$$
$$F(e_3, x_1) = \left\{ \frac{0.65}{m_1}, \frac{0.6}{m_2}, \frac{0.75}{m_3}, \frac{0.9}{m_4}, \frac{0.65}{m_5}, \frac{0.85}{m_6} \right\}$$
$$F(e_3, x_2) = \left\{ \frac{0.6}{m_1}, \frac{0.65}{m_2}, \frac{0.85}{m_3}, \frac{0.9}{m_4}, \frac{0.65}{m_5}, \frac{0.9}{m_6} \right\}$$
$$F(e_3, x_3) = \left\{ \frac{0.5}{m_1}, \frac{0.7}{m_2}, \frac{0.8}{m_3}, \frac{0.95}{m_4}, \frac{0.6}{m_5}, \frac{0.85}{m_6} \right\}$$
$$F(e_4, x_1) = \left\{ \frac{0.5}{m_1}, \frac{0.8}{m_2}, \frac{0.8}{m_3}, \frac{0.95}{m_4}, \frac{0.7}{m_5}, \frac{0.85}{m_6} \right\}$$
$$F(e_4, x_2) = \left\{ \frac{0.55}{m_1}, \frac{0.85}{m_2}, \frac{0.85}{m_3}, \frac{0.95}{m_4}, \frac{0.7}{m_5}, \frac{0.9}{m_6} \right\}$$
$$F(e_4, x_3) = \left\{ \frac{0.65}{m_1}, \frac{0.75}{m_2}, \frac{0.85}{m_3}, \frac{0.9}{m_4}, \frac{0.75}{m_5}, \frac{0.75}{m_6} \right\}$$
$$F(e_5, x_1) = \left\{ \frac{0.7}{m_1}, \frac{0.8}{m_2}, \frac{0.75}{m_3}, \frac{0.9}{m_4}, \frac{0.75}{m_5}, \frac{0.85}{m_6} \right\}$$
$$F(e_5, x_2) = \left\{ \frac{0.75}{m_1}, \frac{0.85}{m_2}, \frac{0.85}{m_3}, \frac{0.85}{m_4}, \frac{0.75}{m_5}, \frac{0.9}{m_6} \right\}$$
$$F(e_5, x_3) = \left\{ \frac{0.8}{m_1}, \frac{0.75}{m_2}, \frac{0.85}{m_3}, \frac{0.9}{m_4}, \frac{0.7}{m_5}, \frac{0.9}{m_6} \right\}$$

Using 4.3, D-fuzzy set for this multi-observer fuzzy soft set, is given by (U, μ) where μ is defined by

$$\mu(m_i) = \frac{1}{|E||X|} \sum_{j=1}^{5} \sum_{k=1}^{3} \mu_{F(e_j, x_k)}(m_i)$$

Hence,

$$\mu(m_1) = \frac{1}{5 \cdot 3}(1.8 + 2.3 + 1.75 + 1.7 + 2.25) = \frac{1}{15}(9.8) = 0.653$$
$$\mu(m_2) = \frac{1}{5 \cdot 3}(2.1 + 2.15 + 1.95 + 2.4 + 2.4) = \frac{1}{15}(11) = 0.733$$
$$\mu(m_3) = \frac{1}{5 \cdot 3}(2.4 + 2.45 + 2.4 + 2.5 + 2.45) = \frac{1}{15}(12.2) = 0.813$$
$$\mu(m_4) = \frac{1}{5 \cdot 3}(2.75 + 2.55 + 2.75 + 2.8 + 2.65) = \frac{1}{15}(13.5) = 0.9$$
$$\mu(m_5) = \frac{1}{5 \cdot 3}(2.1 + 1.9 + 1.9 + 2.15 + 2.2) = \frac{1}{15}(10.25) = 0.683$$
$$\mu(m_6) = \frac{1}{5 \cdot 3}(2.55 + 2.5 + 2.6 + 2.5 + 2.65) = \frac{1}{15}(12.8) = 0.853$$

Since $\mu(m_4) = \max_{1 \le i \le 6} \mu(m_i)$, therefore Mr. Y should purchase the mobile m_4.

4.8. **Application of D-fuzzy set in decision making problem when given information is in form of a multi-observer weighted fuzzy soft set.** Suppose there are three observers $x_1, x_2, x_3 \in X$ in example 3.8 who observe the universe objects.

The corresponding multi-observer weighted fuzzy soft set is given by $(F, A|W)$, where

$$F(e_1|w_1, x_1) = \left\{ \frac{0.8}{c_1}, \frac{0.85}{c_2}, \frac{0.8}{c_3}, \frac{0.75}{c_4}, \frac{0.85}{c_5}, \right\}$$
$$F(e_1|w_1, x_2) = \left\{ \frac{0.75}{c_1}, \frac{0.9}{c_2}, \frac{0.85}{c_3}, \frac{0.8}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_1|w_1, x_3) = \left\{ \frac{0.8}{c_1}, \frac{0.9}{c_2}, \frac{0.75}{c_3}, \frac{0.75}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_2|w_2, x_1) = \left\{ \frac{0.8}{c_1}, \frac{0.7}{c_2}, \frac{0.7}{c_3}, \frac{0.75}{c_4}, \frac{0.95}{c_5}, \right\}$$
$$F(e_2|w_2, x_2) = \left\{ \frac{0.85}{c_1}, \frac{0.75}{c_2}, \frac{0.75}{c_3}, \frac{0.85}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_2|w_2, x_3) = \left\{ \frac{0.8}{c_1}, \frac{0.75}{c_2}, \frac{0.8}{c_3}, \frac{0.85}{c_4}, \frac{0.85}{c_5}, \right\}$$
$$F(e_3|w_3, x_1) = \left\{ \frac{0.7}{c_1}, \frac{0.8}{c_2}, \frac{0.9}{c_3}, \frac{0.9}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_3|w_3, x_2) = \left\{ \frac{0.75}{c_1}, \frac{0.8}{c_2}, \frac{0.9}{c_3}, \frac{0.8}{c_4}, \frac{0.95}{c_5}, \right\}$$
$$F(e_3|w_3, x_3) = \left\{ \frac{0.7}{c_1}, \frac{0.85}{c_2}, \frac{0.9}{c_3}, \frac{0.85}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_4|w_4, x_1) = \left\{ \frac{0.6}{c_1}, \frac{0.9}{c_2}, \frac{0.6}{c_3}, \frac{0.85}{c_4}, \frac{0.85}{c_5}, \right\}$$
$$F(e_4|w_4, x_2) = \left\{ \frac{0.65}{c_1}, \frac{0.8}{c_2}, \frac{0.65}{c_3}, \frac{0.75}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_4|w_4, x_3) = \left\{ \frac{0.7}{c_1}, \frac{0.8}{c_2}, \frac{0.65}{c_3}, \frac{0.8}{c_4}, \frac{0.85}{c_5}, \right\}$$
$$F(e_5|w_5, x_1) = \left\{ \frac{0.8}{c_1}, \frac{0.7}{c_2}, \frac{0.85}{c_3}, \frac{0.75}{c_4}, \frac{0.95}{c_5}, \right\}$$
$$F(e_5|w_5, x_2) = \left\{ \frac{0.85}{c_1}, \frac{0.7}{c_2}, \frac{0.85}{c_3}, \frac{0.75}{c_4}, \frac{0.9}{c_5}, \right\}$$
$$F(e_5|w_5, x_3) = \left\{ \frac{0.85}{c_1}, \frac{0.7}{c_2}, \frac{0.85}{c_3}, \frac{0.8}{c_4}, \frac{0.9}{c_5}, \right\}$$

Using 4.4, D-fuzzy set for this multi-observer weighted fuzzy soft set, is given by (U, μ), where μ is defined by

$$\mu(c_i) = \frac{1}{|E||X|} \sum_{j=1}^{5} \left(\sum_{k=1}^{3} \mu_{F(e_j|w_j, x_k)}(c_i) \right) .w_j$$

Hence,

$$\begin{aligned}
\mu(c_1) &= \frac{1}{5 \cdot 3}(2.35w_1 + 2.45w_2 + 2.15w_3 + 1.95w_4 + 2.5w_5) \\
&= \frac{1}{15}(9.83) = 0.6553 \\
\mu(c_2) &= \frac{1}{5 \cdot 3}(2.65w_1 + 2.2w_2 + 2.45w_3 + 2.5w_4 + 2.1w_5)
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{15}(10.25) = 0.6833\\
\mu(c_3) &= \frac{1}{5\cdot 3}(2.4w_1 + 2.25w_2 + 2.7w_3 + 1.9w_4 + 2.55w_5)\\
&= \frac{1}{15}(10.19) = 0.6793\\
\mu(c_4) &= \frac{1}{5\cdot 3}(2.3w_1 + 2.45w_2 + 2.55w_3 + 2.4w_4 + 2.3w_5)\\
&= \frac{1}{15}(10.3025) = 0.6868\\
\mu(c_5) &= \frac{1}{5\cdot 3}(2.65w_1 + 2.7w_2 + 2.75w_3 + 2.6w_4 + 2.75w_5)\\
&= \frac{1}{15}(11.57) = 0.7713
\end{aligned}$$

Since $\mu(c_5) = \max\limits_{1\le i\le 5}\mu(c_i)$, therefore Mr. Z should buy the car c_5.

5. Conclusion

There are some major advantages of defining and using D-fuzzy set for decision making. The idea of using fuzzy approach instead of non fuzzy approach turns out to be very useful because of the rich theory of fuzzy sets.

The proposed D-fuzzy set gives us the optimal decision when certain information is given in the various forms of soft sets.

The related work in future will be based on more results deduced from D-fuzzy sets obtained from different types of soft sets including multi-observer soft sets having different sets of parameters.

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DEPARTMENT OF MATHEMATICS AND STATISTICS
DEEN DAYAL UPADHYAYA GORAKHPUR UNIVERSITY
273009, GORAKHPUR, INDIA
Email address: mathjulee@gmail.com
Email address: sudhirmaddy149@gmail.com