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Fold thickness of some classes of graphs

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ABSTRACT. A 1-fold of G is the graph G' obtained from a graph G by identifying two nonadjacent vertices in G having at least one common neighbor and reducing the resulting multiple edges to simple edges. A sequence of graphs $G = G_0, G_1, G_2, \ldots, G_k$, where G_{i+1} is a 1-fold of G_i for $i = 0, 1, 2, \ldots, k-1$ is called a uniform k-folding if all the graphs in the sequence are singular or all of them are nonsingular. The largest k for which there exists a uniform k- folding of G is called fold thickness of G and it was first introduced in [Campeña, F. J. H.; Gervacio, S. V. On the fold thickness of graphs. *Arab, J. Math. (Springer)* **9** (2020), no. 2, 345–355]. In this paper, we determine fold thickness of $K_n \odot \overline{K_m}$, $K_n + \overline{K_m}$, cone graph and tadpole graph.

1. INTRODUCTION

The motivation for the concept of graph folding as defined by Gervacio et al. [7] is from the situation of folding a meter stick. Let a finite number of unit bars be joined together at ends in such a way that they are free to turn. There are some meter sticks with this structure as in Fig.1. This meter stick can be considered as a physical model of the path P_n on n vertices and can be folded to become a physical model of the complete graph K_2. It is a natural question to find the minimum fold of a graph, so the concept of fold thickness was introduced as follows.

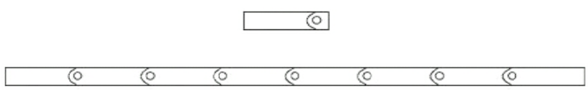

FIGURE 1. Meter stick - Folded and unfolded

Definition 1.1. [2] Let G be a graph that is not isomorphic to a complete graph. If x and y are nonadjacent vertices of G that have atleast one common neighbor, then identify x and y and reduce any resulting multiple edges to simple edges to form a new graph, G'. We call G', a 1-*fold of* G.

Definition 1.2. [2] Consider a sequence of graphs $G = G_0, G_1, G_2, \ldots, G_k$ where G_{i+1} is a 1-fold of G_i for $i = 0, 1, 2, \ldots, k-1$. This sequence is called a k- folding of $G = G_0$.

Let $\mathcal{A}(G_i)$ be the adjacency matrix of G_i. A graph G_i is singular if $\mathcal{A}(G_i)$ is singular and nonsingular if $\mathcal{A}(G_i)$ is nonsingular.

Definition 1.3. [2] A graph G is said to have a uniform k - folding if there is a k- folding in which all graphs in the sequence are singular or all of them are nonsingular. The largest integer k for which there exists a uniform k - folding of G is called *fold thickness* of G, and is denoted by **fold** (G).

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If $G = G_0, G_1, G_2, \ldots, G_k$ is a k-folding of G, the graph G_k is referred as a *k-fold* of G. The *fold thickness* of a graph was first defined by F. J. H. Campeña and S.V. Gervacio in [2] and evaluated fold thickness of some special classes of graphs such as wheel graph, cycle graph, bipartite graphs etc.

2. Preliminary results

In this paper K_n, P_n and C_n denotes the complete graph, path and cycle graph on n vertices respectively. The empty graph $\overline{K_n}$ is the graph with n vertices and zero edges or it is the complement of complete graph, K_n. $V(G)$ and $E(G)$ denotes the vertex set and edge set respectively of a graph G. $\chi(G)$ denotes the vertex chromatic number of G. For any vertex x in a graph G, $N(x)$ is the set of all vertices y in G that are adjacent to x and is called the *neighbor set of x*. Let $G_1, G_2, \ldots, G_n$ be the components of G. Label the vertices of G by labelling the vertices of G_1, then the vertices of G_2 and so on. The adjacency matrix of G, $\mathcal{A}(G)$ is a block diagonal matrix,

$$\mathcal{A}(G) = \begin{bmatrix} \mathcal{A}(G_1) & 0 & \cdots & 0 \\ 0 & \mathcal{A}(G_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathcal{A}(G_n) \end{bmatrix}$$

Thus, the determinant of the adjacency matrix, $\det \mathcal{A}(G) = \prod_{i=1}^{n} \det \mathcal{A}(G_i)$.

The *corona product* [4] $G \odot H$ of two graphs G and H is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining by an edge each vertex from the i^{th}-copy of H with the i^{th}-vertex of G.

The *sum* of two vertex disjoint graphs G and H denoted by $G+H$ is the graph consisting of G and H and all edges of the form xy, where x is a vertex of G and y is a vertex of H.

An *m-gonal n-cone graph*, $C_{m,n}$ is the graph join $C_m + \overline{K_n}$, where C_m is a cycle graph and $\overline{K_n}$ is an empty graph (the graph complement of the complete graph K_n).

The *$(m,n)-$ tadpole graph*, also called a dragon graph or kite graph is the graph obtained by joining a cycle C_m to a path P_n with a bridge.

Theorem 2.1. [3] *Let G be a simple connected graph. The smallest complete graph that G folds into is the complete graph with order $\chi(G)$, where $\chi(G)$ denotes the chromatic number of G.*

Thus, a maximum folding of a graph G on n vertices or simply a max fold of G is defined to be a k-folding of G, where $k = n - \chi(G)$.

Theorem 2.2. [6] *If x and y are vertices in a graph G such that $N(x) = N(y)$, then G is singular.*

Theorem 2.3. [6] *For each $n \geq 1$, $\det \mathcal{A}(K_n) = (-1)^{n-1}(n-1)$.*

Theorem 2.4. [6] *Let x and y be vertices in a graph G such that $N(x) \subseteq N(y)$. If G' is the graph obtained from G by deleting all the edges of the form yz, where z is a neighbor of x, then $\det \mathcal{A}(G) = \det \mathcal{A}(G')$.*

The following theorem gives an upper bound for the fold thickness of graphs.

Theorem 2.5. [2] *For any connected graph G of order n,*

$$\mathbf{fold}(G) \leq \begin{cases} n - \chi(G) & if\ G\ is\ nonsingular, \\ n - \chi(G) - 1 & if\ G\ is\ singular\ . \end{cases}$$

Remark 2.1. In view of the above theorem, if there exists a uniform k-folding of a connected graph G where k is equal to the upper bound in the theorem, then k must be the fold thickness of the graph. This observation will be used to obtain the fold thickness of most of the graphs.

Theorem 2.6. [2] *For each integer $n \geq 1$,*

$$\det \mathcal{A}(P_n) \;=\; \begin{cases} 0 & \text{if } n \text{ is odd,} \\ (-1)^{n/2} & \text{if } n \text{ is even.} \end{cases}$$

Theorem 2.7. [2] *For each integer $n \geq 3$,*

$$\det \mathcal{A}(C_n) \;=\; \begin{cases} 0 & if\ n \equiv 0 \pmod 4, \\ 2 & if\ n \equiv 1\, or\, 3 \pmod 4, \\ -4 & if\ n \equiv 2 \pmod 4. \end{cases}$$

Theorem 2.8. [2] *The path P_n has fold thickness given by,*

$$\textbf{fold}\,(P_n) = \begin{cases} 0 & if\ n\ is\ even \\ \max\{0, n-3\} & if\ n\ is\ odd. \end{cases}$$

Theorem 2.9. [2] *The cycle C_n, has fold thickness given by*

$$\textbf{fold}\,(C_n) = \begin{cases} 0 & if\ n \equiv 2\ (\mathrm{mod}4), \\ n-3 & otherwise. \end{cases}$$

3. Fold thickness of some classes of graphs

3.1. **Corona product, $K_n \odot \overline{K_m}$.** In this section we evaluate the fold thickness of corona product, $K_n \odot \overline{K_m}$ of complete graph K_n and an empty graph $\overline{K_m}$, $m \geq 2$. The vertices of the graph $K_n \odot \overline{K_m}$ is labelled as follows : let $v_1, v_2, \ldots v_n$ be the vertices of K_n and let $u_{i1}, u_{i2}, \ldots u_{im}$ be the pendant vertices adjacent to the i^{th} vertex v_i of K_n for $i = 1, 2, \ldots n$.

Theorem 3.10. *If $m \geq 2$, then the fold thickness of $K_n \odot \overline{K_m}$ is given by,*

$$\textbf{fold}\,(K_n \odot \overline{K_m}) = mn - 2\,.$$

Proof. The graph $K_n \odot \overline{K_m}$, $m \geq 2$ is singular, since the vertices u_{ij} and u_{ik}, where $i \in \{1, 2, \ldots n\}$, $j, k \in \{1, 2, \ldots m\}$ has common neighbor v_i. Therefore, by Theorem 2.5, $\textbf{fold}\,(K_n \odot \overline{K_m}) \leq (m+1)n - \chi(K_n \odot \overline{K_m}) - 1 = (m+1)n - \chi(K_n) - 1 = mn - 1$. For $i = 1, 2, \ldots n-1$, first identify the pendant vertices $u_{i1}, u_{i2}, \ldots u_{im}$ to a single vertex and then identify it with an eligible vertex of K_n. Thus, a uniform $m(n-1)$-folding $G_0 = K_n \odot \overline{K_m}, G_1, \ldots G_{m(n-1)}$ is obtained in which every graph in the sequence is singular and $G_{m(n-1)}$ is the graph K_n plus $m-1$ pendant vertices $u_{n1}, u_{n2}, \ldots u_{nm}$ adjacent to the vertex v_n. Next, identify the vertices $u_{n2}, u_{n3}, \ldots u_{nm}$ of $G_{m(n-1)}$ one by one to obtain a graph G' which is K_n plus two pendant vertices adjacent to the vertex v_n. Thus, a uniform $(m-2)$-folding of $G_{m(n-1)}$ is obtained in which each graph is singular. If the two pendant vertices of G' are identified, then we obtain a nonsingular graph which is K_n plus one pendant vertex adjacent to one of its vertices. Thus, $\textbf{fold}\,(K_n \odot \overline{K_m}) = m(n-1) + m - 2 = mn - 2$. □

3.2. **Sum graph, $K_n + \overline{K_m}$.** In this section, we evaluate the fold thickness of sum of K_n and $\overline{K_m}$, $K_n + \overline{K_m}$. Let $u_1, u_2, \ldots u_n$ be the vertices of K_n and $v_1, v_2, \ldots v_m$ be the vertices of $\overline{K_m}$.

Theorem 3.11. *If $m \geq 2$, then the fold thickness of $K_n + \overline{K_m}$ is given by,*

$$\textbf{fold}\,(K_n + \overline{K_m}) = m - 2.$$

Proof. The graph $K_n + \overline{K_m}$, $m \geq 2$ is singular since, for any two vertices x and y in $V(\overline{K_m})$, $N(x) = N(y) = V(K_n)$. Note that $\chi(K_n + \overline{K_m}) = \chi(K_n) + 1 = n + 1$. By Theorem 2.5, $\textbf{fold}\,(K_n + \overline{K_m}) \leq m + n - \chi(K_n + \overline{K_m}) - 1 = m + n - (n+1) - 1 = m - 2$. Identify the vertices $v_2, v_3, \ldots v_m$ to a single vertex. So, a uniform $m - 2$ - folding $G_0 = K_n + \overline{K_m}, G_1, \ldots G_{m-2}$ is obtained in which the last graph G_{m-2} is isomorphic to the graph $K_n + \overline{K_2}$. If the vertices of $\overline{K_2}$ are identified, then a complete graph on $n + 1$ vertices is obtained and is nonsingular. Hence, $\textbf{fold}\,(K_n + \overline{K_m}) = m - 2$. $\square$

3.3. **Cone graph, $C_{m,n}$.** In this section, we determine the fold thickness of cone graph $C_{m,n}$. $C_{m,1}$ is the wheel graph and its fold thickness is studied in [2]. Hence, fold thickness for the case $n \geq 2$ is evaluated in this section.

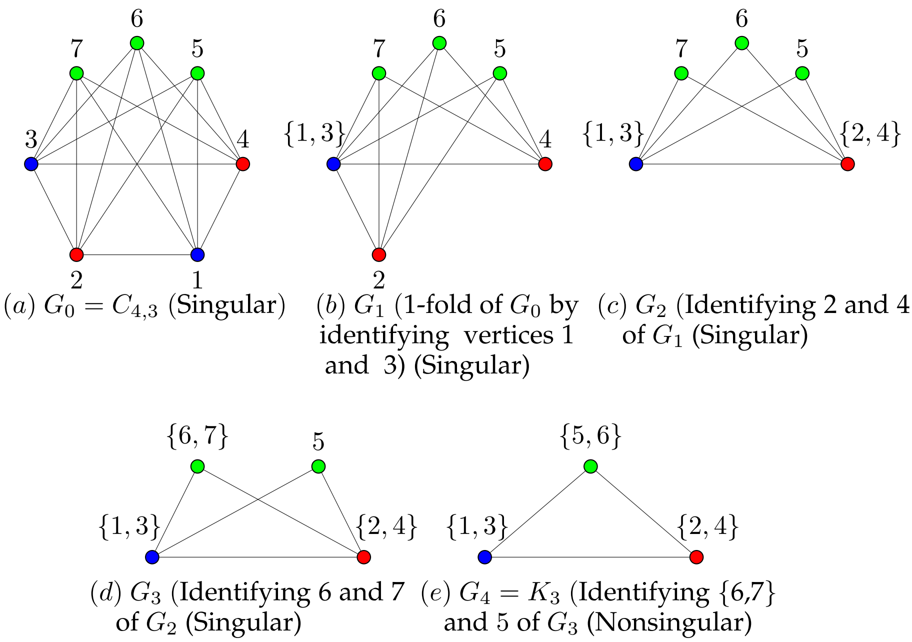
Theorem 3.12. *For $m \geq 3$ and $n \geq 2$, the fold thickness of cone graph $C_{m,n}$ is given by,*

$$\textbf{fold}\,(C_{m,n}) = \begin{cases} m + n - 5 & if\ m\ is\ odd \\ m + n - 4 & if\ m\ is\ even. \end{cases}$$

Proof. By definition of cone graph, if $x, y \in V(\overline{K_n})$, $N(x) = N(y) = V(C_m)$. Hence, by Theorem 2.2, $C_{m,n}$, $n \geq 2$ is singular. Clearly, $\chi(C_{m,n}) = \chi(C_m) + 1$. So, $\chi(C_{m,n}) = 3$, if m is even and $\chi(C_{m,n}) = 4$, if m is odd. Since $N(x) = N(y)$ for any pair of vertices x and y in $V(\overline{K_n})$, folding C_m in any manner keeps $C_{m,n}$ singular. So, we have to find the maximum folding of C_m.
Case 1 : *m is odd.*

By Theorem 2.5, $\textbf{fold}(C_{m,n}) \leq m + n - 4 - 1 = m + n - 5$. Since $\chi\,(C_m) = 3$, there exists an $(m - 3)$- maximum folding of C_m, by Theorem 2.1. After these $(m - 3)$ foldings, C_m becomes K_3 and $C_{m,n}$ becomes $K_3 + \overline{K_n}$, a singular graph. Fold $K_3 + \overline{K_n}$ by identifying any two vertices of $\overline{K_n}$. Continue this process $(n-2)$ times until there remains exactly two vertices in $\overline{K_n}$. The graph now obtained is $K_3 + \overline{K_2}$. In this $(n - 2)$-folding, all graphs are singular. If the remaining two vertices of $\overline{K_2}$ in $K_3 + \overline{K_2}$ are identified, we obtain the graph $K_3 + K_1$, which is the complete graph K_4 and is nonsingular. So, the maximum folding of $K_3 + \overline{K_n}$ in which all graphs are singular is $(n-2)$. Hence, an $(m-3)+(n-2) = (m+n-5)$- uniform folding of $C_{m,n}$ is obtained. Consequently, $\textbf{fold}(C_{m,n}) = m + n - 5$.
Case 2 : *m is even.*

By Theorem 2.5, $\textbf{fold}(C_{m,n}) \leq m + n - 3 - 1 = m + n - 4$. There exists an $(m - 2)$- maximum folding of C_m, since $\chi\,(C_m) = 2$. After these $(m - 2)$ foldings, C_m becomes K_2 and $C_{m,n}$ becomes $K_2 + \overline{K_n}$, a singular graph. Similarly, fold $K_2 + \overline{K_n}$ as in the above case, $(n - 2)$ times by identifying vertices of $\overline{K_n}$ until the singular graph $K_2 + \overline{K_2}$ is obtained. All graphs in this $(n - 2)$- folding are singular. If the remaining two vertices in $\overline{K_2}$ of $K_2 + \overline{K_2}$ are identified, then we obtain $K_2 + K_1$, which is the complete graph K_3 and is nonsingular. Thus, maximum folding of $K_2 + \overline{K_n}$ in which all graphs are singular is $(n - 2)$. Hence, an $(m - 2) + (n - 2) = (m + n - 4)$- uniform folding of $C_{m,n}$ is obtained. So, $\textbf{fold}(C_m) = m + n - 4$. $\square$



(a) $G_0 = C_{4,3}$ (Singular) (b) G_1 (1-fold of G_0 by identifying vertices 1 and 3) (Singular) (c) G_2 (Identifying 2 and 4 of G_1 (Singular)

(d) G_3 (Identifying 6 and 7 of G_2 (Singular) (e) $G_4 = K_3$ (Identifying {6,7} and 5 of G_3 (Nonsingular)

FIGURE 2. Folding process of the cone graph $C_{4,3}$

An example of the folding process for the case $m = 4$ and $n = 3$ is shown in Fig.2. Two vertices having distinct colors cannot be identified, since they are adjacent. We can observe that **fold**$(C_{4,3}) = 3$.

3.4. **Tadpole graph.** Here, we evaluate the fold thickness of the tadpole graph $T_{m,n}$.

Lemma 3.1. *For $m \geq 3$ and $n \geq 1$,*

$$\det \mathcal{A}(T_{m,n}) = \begin{cases} (-1)^{(n-1)/2} \det \mathcal{A}(P_{m+1}) & if\ n\ is\ odd \\ (-1)^{n/2} \det \mathcal{A}(C_m) & if\ n\ is\ even. \end{cases}$$

Proof. Let x, y and z be vertices of $T_{m,n}$, where x is the pendant vertex, y is the unique neighbor of x and z is a vertex adjacent to y. In $T_{m,1}$, apply Theorem 2.4 and remove the edge yz to obtain the graph P_{m+1}. Hence, $\det \mathcal{A}(T_{m,1}) = \det \mathcal{A}(P_{m+1})$. Similarly, in $T_{m,2}$, the edge yz can be removed to obtain a disconnected graph with two components, K_2 and C_m such that $\det \mathcal{A}(T_{m,2}) = \det \mathcal{A}(K_2) \det \mathcal{A}(C_m) = (-1) \det \mathcal{A}(C_m)$. If $n \geq 3$, by Theorem 2.4, we get $\det \mathcal{A}(T_{m,n}) = \det \mathcal{A}(T_{m,n-2}) \det \mathcal{A}(K_2)$. Then, by applying mathematical induction on n, we can conclude that $\det \mathcal{A}(T_{m,n}) = (-1)^{(n-1)/2} \det \mathcal{A}(P_{m+1})$ if n is odd and $\det \mathcal{A}(T_{m,n}) = (-1)^{n/2} \det \mathcal{A}(C_m)$ if n is even. □

Remark 3.2. By above lemma, we can observe that, if m is odd, $T_{m,n}$ is nonsingular for all $n \geq 1$. If $m \equiv 0 \pmod 4$, $T_{m,n}$ is singular for all $n \geq 1$. If $m \equiv 2 \pmod 4$, $T_{m,n}$ is singular for odd n and nonsingular for even n.

Theorem 3.13. *For $m \geq 3$ and $n \geq 1$,*

$$\textbf{fold}\,(T_{m,n}) = \begin{cases} 0 & if\ m\ \equiv 2 \pmod 4\ and\ n\ is\ even \\ m+n-3 & otherwise. \end{cases}$$

Proof. The chromatic number of $T_{m,n}$ and C_m are the same. So, $\chi(T_{m,n}) = 2$, if m is even and $\chi(T_{m,n}) = 3$, if m is odd.

Case 1 : *m is odd.*

By Lemma 3.1, $T_{m,n}$ is non-singular for $n \geq 1$. Therefore, by Theorem 2.5, **fold** $(T_{m,n}) \leq m+n-3$. Consider the n-uniform folding $G_0, G_1, \ldots, G_n$ of $T_{m,n}$, where G_{i+1} is a 1- fold of G_i for $i = 1, 2, \ldots, n-1$, obtained by identifying the pendant vertex of G_i with an eligible vertex. Each graph G_i , $i = 1, 2, \ldots, n-1$ is nonsingular by Lemma 3.1 and G_n is the cycle C_m. Since **fold** $(C_m) = m-3$, an $n + (m-3) = (m+n-3)$- uniform folding of $T_{m,n}$ is obtained in which all graphs are nonsingular. Thus, **fold** $(T_{m,n}) = m+n-3$.

Case 2 : $m \equiv 0 \,(\mathrm{mod}\ 4)$

Here, $T_{m,n}$, $n \geq 1$ is singular. So, by Theorem 2.5, **fold** $(T_{m,n}) \leq m+n-2-1 = m+n-3$. Fold this graph n times as in the above case by identifying the pendant vertex with the eligible vertex on each step. Then, we obtain an n- uniform folding $G_0, G_1, \ldots, G_n$, where G_n is C_m. By Lemma 3.1, each graph in this sequence is singular. Since **fold** $(C_m) = m-3$, we get an $(m+n-3)$- uniform folding of $T_{m,n}$ and hence **fold** $(T_{m,n}) = m+n-3$.

Case 3 : $m \equiv 2 \,(\mathrm{mod}\ 4)$

By Lemma 3.1, the graph $T_{m,n}$ is singular if n is odd and nonsingular if n is even.

If n is odd, **fold** $(T_{m,n}) \leq m+n-2-1 = m+n-3$ by Theorem 2.5. Fold the graph $T_{m,n}$ once as in Fig 3 to obtain the graph $T_{m-2,n+1}$. Since, $m-2 \equiv 0 \,(\mathrm{mod}\ 4)$, the graph $T_{m-2,n+1}$ is singular. From Case 2, **fold** $(T_{m-2,n+1}) = (m-2)+(n+1)-3 = m+n-4$. Therefore, we get a $1 + (m+n-4) = m+n-3$ - uniform folding of $T_{m,n}$. Hence, **fold** $(T_{m,n}) = m+n-3$.

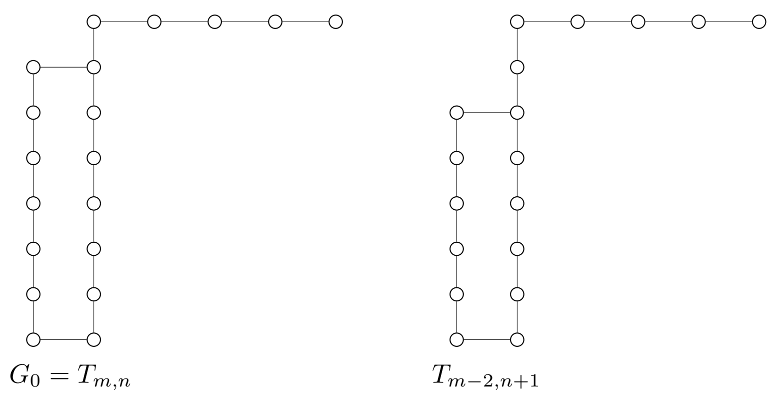


FIGURE 3. A uniform 1-fold of $T_{m,n}$ when $m \equiv 2 \,(\mathrm{mod}\ 4)$ and n is odd.

When n is even, obtain a 1-fold of $T_{m,n}$ by identifying two eligible vertices. There are three possibilities. Both vertices are in $V(P_n)$, one is in $V(P_n)$ and other is in $V(C_m)$ or both are in $V(C_m)$. In any case, one of the following graphs are obtained: (1) A graph which has at least two distinct vertices having a common neighbor (2) A graph, after applying Theorem 2.4 sufficient number of times by taking x as the pendant vertex and y as a vertex adjacent to the unique neighbor of x, leads to a disconnected graph in which one component is $T_{m,1}$ (3) A graph, after applying Theorem 2.4 sufficient number of times by taking x as the pendant vertex and y as a vertex adjacent to the unique neighbor of x, leads to a disconnected graph in which one component has at least two distinct vertices having a common neighbor. By Theorem 2.2 and Remark 3.2, all these graphs are singular. This implies that, any 1-fold of $T_{m,n}$ is a singular graph. Hence, **fold** $(T_{m,n}) = 0$. □

The m- pan graph is the graph obtained by joining a cycle graph C_m to K_1 with a bridge and hence it is isomorphic to the $(m, 1)$- tadpole graph.

Corollary 3.1. *The fold thickness of m - pan graph is given by* **fold** $(T_{m,1}) = m - 2$.

Proof. The m- pan graph is the special case of tadpole graph $T_{m,n}$ when $n = 1$. By Theorem 3.13, the conclusion follows easily. □

4. Conclusion

In this paper, we have determined the fold thickness of $K_n \odot \overline{K_m}$, $K_n + \overline{K_m}$, cone graph and tadpole graph. It can be extended to various classes of graphs such as product graphs, bipartite graphs and in general to any incomplete graph. This also gives a lot of problems for further research such as characterizing graphs with fold thickness as a parameter.

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