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On the Quinary Fibonacci-Padovan Sequences

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ABSTRACT. In this paper, we consider the Fibonacci and Padovan sequences. We introduce the quinary Fibonacci-Padovan sequences whose compounds are the Fibonacci and Padovan sequences. We derive the Binet-like formulas, the generating functions and exponential generating functions of these sequences. Also, we obtain some binomial identities, series and sums for them.

1. INTRODUCTION

Special numbers and the corresponding recurrence relations and their generalizations have many applications to every field of science and they have many interesting properties [17–19]. Second order linear recurrences related to Fibonacci and Lucas numbers and their generalizations are investigated in [8, 14, 15, 21]. Fourth order linear recurrences and their generalizations are studied in [9, 10, 20, 23]. On algebraic identities on a new integer sequence with four parameters are investigated in [24].

In [3] the authors studied on the quadra Fibona-Pell and hexa Fibona-Pell-Jacobsthal sequences.

In [9] and [10] various fourth order linear recurrences and their polynomials are defined and studied.

In [23] the author define the quadrapell numbers and quadrapell polynomials as fourth order linear recurrences.

In [20] the author defines the quadra Fibona-Pell integers sequences and she gives some algebraic identities.

In the present work we consider fifth orders linear recurrences and we define the quinary Fibonacci-Padovan sequences. We give some properties of them.

The Fibonacci and Padovan sequences $\{F_n\}$ and $\{P_n\}$ are defined by second and third order recurrences for $n \geq 0$, respectively,

$$F_{n+2} = F_{n+1} + F_n,$$

$$P_{n+3} = P_{n+1} + P_n,$$

with the initial conditions are given as follow, respectively,

$$F_0 = 0, \quad \text{and} \quad F_1 = 1,$$

$$P_0 = 1, \quad P_1 = 1 \quad \text{and} \quad P_2 = 1,$$

The first few members of these sequences are given as follow, respectively,

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n	0	1	2	3	4	5	6	7	8	9	10	11	...
F_n	0	1	1	2	3	5	8	13	21	34	55	89	...
P_n	1	1	1	2	2	3	4	5	7	9	12	16	...

TABLE 1. The first few members of these sequences

The recurrences involve the characteristic equations, respectively,

$$x^2 - x - 1 = 0,$$

$$y^3 - y - 1 = 0,$$

The roots of the equations are as follows, respectively,

$$\alpha = \frac{1+\sqrt{5}}{2} \quad \text{and} \quad \beta = \frac{1-\sqrt{5}}{2}, \tag{1.1}$$

$$\lambda = \sqrt[3]{\frac{9+\sqrt{69}}{18}} + \sqrt[3]{\frac{9-\sqrt{69}}{18}} = p = Plastic \quad ratio \tag{1.2}$$

$$\delta = -\sqrt[3]{\frac{9+\sqrt{69}}{144}} - \sqrt[3]{\frac{9-\sqrt{69}}{144}} + i\frac{\sqrt{3}}{2}\left(\sqrt[3]{\frac{9+\sqrt{69}}{18}} - \sqrt[3]{\frac{9-\sqrt{69}}{18}}\right)$$

$$\gamma = -\sqrt[3]{\frac{9+\sqrt{69}}{144}} - \sqrt[3]{\frac{9-\sqrt{69}}{144}} - i\frac{\sqrt{3}}{2}\left(\sqrt[3]{\frac{9+\sqrt{69}}{18}} - \sqrt[3]{\frac{9-\sqrt{69}}{18}}\right).$$

Then the following equalities follow directly from Vieta's formulas, respectively,

$$\alpha+\beta = 1, \quad \alpha-\beta = \sqrt{5}, \quad \alpha\beta = -1,$$

$$\lambda+\delta+\gamma = 0, \quad \lambda\delta+\lambda\gamma+\delta\gamma = -1, \quad \lambda\delta\gamma = 1.$$

Moreover, the Binet and Binet-like formulas for the Fibonacci and Padovan sequences are, respectively,

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta},$$

$$P_n = a\lambda^n + b\delta^n + c\gamma^n,$$

where

$$a = \frac{(\delta-1)(\gamma-1)}{(\lambda-\delta)(\lambda-\gamma)}, \quad b = \frac{(\lambda-1)(\gamma-1)}{(\delta-\lambda)(\delta-\gamma)} \quad \text{and} \quad c = \frac{(\lambda-1)(\delta-1)}{(\gamma-\lambda)(\gamma-\delta)}.$$

The generating functions for the Fibonacci and Padovan sequences are, respectively,

$$G_F(x) = \sum_{n=0}^{\infty} F_n x^n = \frac{x}{1-x-x^2},$$

$$G_P(x) = \sum_{n=0}^{\infty} P_n x^n = \frac{1+x}{1-x^2-x^3}.$$

The exponential generating functions for the Fibonacci and Padovan sequences are, respectively,

$$E_F(x) = \sum_{n=0}^{\infty} \frac{F_n}{n!} x^n = \frac{e^{\alpha x} - e^{\beta x}}{\alpha - \beta},$$

$$E_P(x) = \sum_{n=0}^{\infty} \frac{P_n}{n!} x^n = ae^{\lambda x} + be^{\delta x} + ce^{\gamma x}.$$

The series for the Fibonacci and Padovan sequences are, respectively,

$$S_F(x) = \sum_{n=0}^{\infty} \frac{F_n}{x^n} = \frac{x}{x^2 - x - 1},$$

$$S_P(x) = \sum_{n=0}^{\infty} \frac{P_n}{x^n} = \frac{x^2(x+1)}{x^3 - x - 1}.$$

The sum formulas for the Fibonacci and Padovan sequences are, respectively,

$$T_F(x) = \sum_{n=0}^{m} F_n = F_{m+2} - 1,$$

$$T_P(x) = \sum_{n=0}^{m} P_n = P_{m+5} - 2.$$

The Fibonacci and Padovan sequences and identities in the above passage are available in [1–6, 11–13, 17, 18, 22, 25].

2. On the Quinary Fibonacci-Padovan Sequences

In this section, we aim to obtain a new sequence whose characteristic equation has the same roots as the characteristic equations of the Fibonacci and Padovan sequences. Then we will examine the situation of these new sequences in the initial conditions, find the Binet-like formula and reach the generating function, series, sum and binomial sum. Similar investigations were given in [16, 20, 23]. In [23] the quadra Pell numbers are defined and some properties are given. In [20] the Fibona-Pell integer sequence is defined and some algebraic identities are obtained. In [16] the Quadra Lucas-Jacobsthal Numbers were investigated.

Definition 2.1. The quinary Fibonacci-Padovan (or Fibona-Padovan) sequence $\{FP_n\}_{n\geq 0}$ is defined by a fifth order recurrence;

$$FP_{n+5} = FP_{n+4} + 2FP_{n+3} - 2FP_{n+1} - FP_n \tag{2.3}$$

with the initial conditions $FP_0 = 0$, $FP_1 = 1$, $FP_2 = 2$, $FP_3 = 4$ and $FP_4 = 8$.

A similar definition has been made in [7]. The first few members of this sequence are given as follow

n	0	1	2	3	4	5	6	7	8	9	10	...
FP_n	0	1	2	4	8	14	25	43	73	123	205	...

Table 2. The first few members of the quinary Fibonacci-Padovan sequence

We note that in Definition 2.1, if we take the initial conditions as follows we generate the Fibonacci and Padovan numbers.

n	0	1	2	3	4	Numbers
FP_n	0	1	1	2	3	Fibonacci numbers
FP_n	1	1	1	2	2	Padovan numbers

TABLE 3. The first few members with the different initial conditions

The characteristic equation associated to the recurrence relation (2.3) is

$$z^5 - z^4 - 2z^3 + 2z + 1 = 0. \tag{2.4}$$

The roots of the equation (2.4) are the root in (1.1) and (1.2). Then the following equalities follow directly from Vieta's formulas

$$\alpha + \beta + \lambda + \delta + \gamma = 1 \quad \text{and} \quad \alpha\beta\lambda\delta\gamma = -1.$$

Theorem 2.1. *The Binet-like formula for the quinary Fibonacci-Padovan sequence is*

$$FP_n = a_1\alpha^n + a_2\beta^n + a_3\lambda^n + a_4\delta^n + a_5\gamma^n,$$

where,

$$\begin{aligned}
a_1 &= \frac{3\alpha+2}{(\alpha-\beta)(\alpha-\lambda)(\alpha-\delta)(\alpha-\gamma)},\\
a_2 &= \frac{3\beta+2}{(\beta-\alpha)(\beta-\lambda)(\beta-\delta)(\beta-\gamma)},\\
a_3 &= \frac{\lambda+\delta\gamma+3}{(\lambda-\alpha)(\lambda-\beta)(\lambda-\delta)(\lambda-\gamma)},\\
a_4 &= \frac{\delta+\lambda\gamma+3}{(\delta-\alpha)(\delta-\beta)(\delta-\lambda)(\delta-\gamma)},\\
a_5 &= \frac{\gamma+\lambda\delta+3}{(\gamma-\alpha)(\gamma-\beta)(\gamma-\lambda)(\gamma-\delta)}.
\end{aligned}$$

such that α, β, λ, δ and γ are the roots of the characteristic equation of the quinary Fibonacci-Padovan sequence.

Proof.

$$\begin{aligned}
F_0 &= a_1 + a_2 + a_3 + a_4 + a_5 = 0,\\
F_1 &= a_1\alpha + a_2\beta + a_3\lambda + a_4\delta + a_5\gamma = 1,\\
F_2 &= a_1\alpha^2 + a_2\beta^2 + a_3\lambda^2 + a_4\delta^2 + a_5\gamma^2 = 2,\\
F_3 &= a_1\alpha^3 + a_2\beta^3 + a_3\lambda^3 + a_4\delta^3 + a_5\gamma^3 = 4,\\
F_4 &= a_1\alpha^4 + a_2\beta^4 + a_3\lambda^4 + a_4\delta^4 + a_5\gamma^4 = 8.
\end{aligned}$$

By the Cramer's rule or the Gauss-Jordan elimination method, we can easily compute the coefficients a_i's of the system of linear equations above this completes the proof.

□

Theorem 2.2. *The generating function for the quinary Fibonacci-Padovan sequence is*

$$G_{FP}(x) = \sum_{n=0}^{\infty} FP_n x^n = \frac{x^2+x}{1-x-2x^2+2x^4+x^5}.$$

Proof. Let

$$G_{FP}(x) = \sum_{n=0}^{\infty} FP_n x^n = FP_0 + FP_1 x + FP_2 x^2 + FP_3 x^3 + \cdots + FP_n x^n + \ldots$$

be the generating function of the quinary Fibonacci-Padovan sequence. Multiply both of side of the equality by the term $-x$, $-2x^2$ $2x^4$ and x^5, respectively, such as

$$\begin{aligned}
-xG_{FP}(x) &= -FP_0 x - FP_1 x^2 - FP_2 x^3 - FP_3 x^4 - \cdots - FP_n x^{n+1} - \ldots \\
-2x^2 G_{FP}(x) &= -2FP_0 x^2 - 2FP_1 x^3 - 2FP_2 x^4 - 2FP_3 x^5 - \cdots - 2FP_n x^{n+2} - \ldots \\
2x^4 G_{FP}(x) &= 2FP_0 x^4 + 2FP_1 x^5 + 2FP_2 x^6 + 2FP_3 x^7 + \cdots + 2FP_n x^{n+4} + \ldots \\
x^5 G_{FP}(x) &= FP_0 x^5 + FP_1 x^6 + FP_2 x^7 + FP_3 x^8 + \cdots + FP_n x^{n+5} + \ldots
\end{aligned}$$

Then, we write

$$\begin{aligned}
(1 - x - 2x^2 + 2x^4 + x^5)G_{FP}(x) = FP_0 &+ (FP_1 - FP_0)x + (FP_2 - FP_1 - 2FP_0)x^2 \\
&+ (FP_3 - FP_2 - 2FP_1)x^3 + (FP_4 - FP_3 - 2FP_2 + 2FP_0)x^4 \\
&+ (FP_5 - FP_4 - 2FP_3 + 2FP_1 + FP_0)x^5 \ldots \\
&+ (FP_n - FP_{n-1} - 2FP_{n-2} + 2FP_{n-4} + FP_{n-5})x^n + \ldots
\end{aligned}$$

Now, by using the initial conditions of the quinary Fibonacci-Padovan sequence and

$$FP_n - FP_{n-1} - 2FP_{n-2} + 2FP_{n-4} + FP_{n-5} = 0,$$

we obtain that

$$G_{FP}(x) = \sum_{n=0}^{\infty} FP_n x^n = \frac{x^2 + x}{1 - x - 2x^2 + 2x^4 + x^5}.$$

Thus, the proof is completed. □

The generating function of the quinary Fibonacci-Padovan sequence is the multiplication of the generating function of the Fibonacci and Padovan sequence as seen following,

$$G_F(x)G_P(x) = \left(\frac{x}{1 - x - x^2}\right)\left(\frac{1 + x}{1 - x - x^3}\right) = \frac{x^2 + x}{x^5 + 2x^4 - 2x^2 - x + 1} = G_{FP}(x)$$

Theorem 2.3. *The exponential generating function for the quinary Fibonacci-Padovan sequence is*

$$E_{FP}(x) = a_1 e^{\alpha x} + a_2 e^{\beta x} + a_3 e^{\lambda x} + a_4 e^{\delta x} + a_5 e^{\gamma x} = \sum_{n=0}^{\infty} \frac{FP_n}{n!} x^n.$$

Proof. We know that,

$$e^{\alpha x} = \sum_{n=0}^{\infty} \frac{\alpha^n x^n}{n!}, \quad e^{\beta x} = \sum_{n=0}^{\infty} \frac{\beta^n x^n}{n!}, \quad e^{\lambda x} = \sum_{n=0}^{\infty} \frac{\lambda^n x^n}{n!}, e^{\delta x} = \sum_{n=0}^{\infty} \frac{\delta^n x^n}{n!} \quad \text{and} \quad e^{\gamma x} = \sum_{n=0}^{\infty} \frac{\gamma^n x^n}{n!}$$

Multiplying each side of the identities, respectively, by a_1, a_2, a_3, a_4 and a_5 and adding of them, we obtain that

$$\begin{aligned}
E_{FP}(x) &= a_1 e^{\alpha x} + a_2 e^{\beta x} + a_3 e^{\lambda x} + a_4 e^{\delta x} + a_5 e^{\gamma x} \\
&= \sum_{n=0}^{\infty} (a_1 \alpha^n + a_2 \beta^n + a_3 \lambda^n + a_4 \delta^n + a_5 \gamma^n) \frac{1}{n!} x^n \\
&= \sum_{n=0}^{\infty} \frac{FP_n}{n!} x^n.
\end{aligned}$$

□

Theorem 2.4. *The series for the quinary Fibonacci-Padovan sequence is*

$$S_{FP}(x)=\sum_{n=0}^{\infty}\frac{FP_n}{x^n}=\frac{x^4+x^3}{x^5-x^4-2x^3+2x+1}.$$

Proof. Let

$$S_{FP}(x)=\sum_{n=0}^{\infty}\frac{FP_n}{x^n}=FP_0+\frac{FP_1}{x}+\frac{FP_2}{x^2}+\frac{FP_3}{x^3}+\cdots+\frac{FP_n}{x^n}+\ldots$$

be the series of the quinary Fibonacci-Padovan sequence. Multiply both of side of the equality by the term x^5, $-x^4$ $-2x^3$ and $2x$ respectively, such as

$$\begin{aligned}
x^5S_{FP}(x)&=x^5FP_0+x^4FP_1+x^3FP_2+x^2FP_3+\cdots+\frac{FP_n}{x^{n-5}}+\ldots\\
-x^4S_{FP}(x)&=-x^4FP_0-x^3FP_1-x^2FP_2-xFP_3-\cdots-\frac{FP_n}{x^{n-4}}-\ldots\\
-2x^3S_{FP}(x)&=-2x^3FP_0-2x^2FP_1-2xFP_2-2FP_3-\cdots-2\frac{FP_n}{x^{n-3}}-\ldots\\
2xS_{FP}(x)&=2xFP_0+2FP_1+2\frac{FP_2}{x}+2\frac{FP_3}{x^2}+\cdots+2\frac{FP_n}{x^{n-1}}+\ldots
\end{aligned}$$

Then, we write

$$\begin{aligned}
(x^5-x^4-2x^3+2x+1)S_{FP}(x)&=FP_0+(FP_1-FP_0)x^4+(FP_2-FP_1-2FP_0)x^3\\
&+(FP_3-FP_2-2FP_1)x^2+(FP_4-FP_3-2FP_2+2FP_0)x\\
&+(FP_5-FP_4-2FP_3+2FP_1+FP_0)+\ldots\\
&+(FP_n-FP_{n-1}-2FP_{n-2}+2FP_{n-4}+FP_{n-5})\frac{1}{x^{n-5}}+\ldots
\end{aligned}$$

Now, by using the initial conditions of the quinary Fibonacci-Padovan sequence and

$$FP_n-FP_{n-1}-2FP_{n-2}+2FP_{n-4}+FP_{n-5}=0,$$

we obtain that

$$S_{FP}(x)=\sum_{n=0}^{\infty}\frac{FP_n}{x^n}=\frac{x^4+x^3}{x^5-x^4-2x^3+2x+1}.$$

Thus, the proof is completed. □

The series for the quinary Fibonacci-Padovan sequence is the multiplication of the series for the Fibonacci and Padovan sequence as seen following,

$$S_F(x)S_P(x)=\left(\frac{x}{x^2-x-1}\right)\left(\frac{x^3+x^2}{x^3-x-1}\right)=\frac{x^4+x^3}{x^5-x^4-2x^3+2x+1}=S_{FP}(x)$$

Theorem 2.5. *The sum of the first n terms of FP_n is*

$$\sum_{i=0}^{n}FP_i=2FP_{n+3}+2FP_{n+2}-FP_{n+5}+2,\quad n\geq 0.$$

Proof. We know that

$$FP_{n+5} = FP_{n+4} + 2FP_{n+3} - 2FP_{n+1} - FP_n$$

So,

$$FP_{n+5} - FP_{n+4} = 2FP_{n+3} - 2FP_{n+1} - FP_n$$

Applying to the identity above, we deduce that

$$\begin{aligned} FP_5 - FP_4 &= 2FP_3 - 2FP_1 - FP_0, \\ FP_6 - FP_5 &= 2FP_4 - 2FP_2 - FP_1, \\ FP_7 - FP_6 &= 2FP_5 - 2FP_3 - FP_2, \\ &\dots, \\ FP_{n+3} - FP_{n+2} &= 2FP_{n+1} - 2FP_{n-1} - FP_{n-2}, \\ FP_{n+4} - FP_{n+3} &= 2FP_{n+2} - 2FP_n - FP_{n-1}, \\ FP_{n+5} - FP_{n+4} &= 2FP_{n+3} - 2FP_{n+1} - FP_n \end{aligned}$$

If we sum of both of sides of the identities above, we obtain,

$$FP_{n+5} - FP_4 = 2FP_{n+3} + 2FP_{n+2} - 2FP_2 - 2FP_1 - \sum_{i=0}^{n} FP_i.$$

Hence, we get the desired result. □

3. Conclusion

In this paper, we define a new compound sequence as the Fibonacci-Padovan sequence. We prove that their characteristic equation is a multiplication of the characteristic equations of Fibonacci and Padovan. We show that by certain initial conditions from these sequences we derive all of the compound sequences: the Fibonacci and Padovan. We give the Binet-like formula for quinary Fibonacci-Padovan sequence. Finally, we obtain their generating functions, series and sums. Also, we see that the generating functions of these sequences arise from the multiplication of the generating functions of Fibonacci and Padovan sequences.

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