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Convergence Results for Mean Nonexpansive Mappings in Uniformly Convex Banach Space

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ABSTRACT. In this paper, we establish weak and strong convergence theorems for mean nonexpansive mappings in Banach space under the M-iteration process.

1. INTRODUCTION

Let Y be a Banach Space and $\emptyset \neq W \subseteq Y$. Consider the selfmap $S : W \to W$. An element $e_0 \in W$ is called a fixed point if

$$Se_0 = e_0$$

The set of all fixed points of S is denoted by $F(S)$. Fixed point theory has always an important role in the field of analysis. The Banach contraction principle is a milestone in the fixed point theory. It always inspires researchers to obtain fixed points of different mappings over different spaces. Fixed point problems possess either an existing problem or an approximate scheme. Over the years, researchers have developed many iterative schemes to obtain approximate solutions to fixed point problems for different mappings or operators over different spaces. In 1953 Mann [10] introduced an iterative scheme. Let $S : W \to W$ be any nonlinear mapping. For each $p_0 \in W$, the sequence $\{p_n\}$ in W is defined by

$$\begin{cases} p_0 \in W \\ p_{n+1} = (1-\alpha_n)p_n + \alpha_n S p_n. \end{cases} \tag{1.1}$$

where $\alpha_n \in [0, 1]$.
Later Agarwal et al.[1] introduced an iteration process, also called $S - iteration$ process, is defined as: Let $S : W \to W$ be any nonlinear mapping. For each $p_0 \in W$, the sequence $\{p_n\}$ in W is defined by

$$\begin{cases} p_0 \in W \\ q_n = (1-\beta_n)p_n + \beta_n S p_n \\ p_{n+1} = (1-\alpha_n)Sp_n + \alpha_n S q_n. \end{cases} \tag{1.2}$$

where $\alpha_n, \beta_n \in [0, 1]$
2014, Gursoy and Karakaya [7] introduced a new iteration process called the Picard-S iteration process. Let $S : W \to W$ be any nonlinear mapping. For each $p_0 \in W$, the

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sequence $\{p_n\}$ in W is defined by

$$\begin{cases} p_0 \in W \\ r_n = (1-\beta_n)p_n + \beta_n S p_n \\ q_n = (1-\alpha_n)Sp_n + \alpha_n S r_n \\ p_{n+1} = S q_n \end{cases} \tag{1.3}$$

where $\alpha_n, \beta_n \in [0,1]$. They proved that the Picard-S iteration process converges faster than all Picard, Mann, and Ishikawa iteration Processes.
In 2015, Thakur et, al.[15] used a new iteration process, defined as; let $S : W \to W$ be any nonlinear mapping. For each $p_0 \in W$, the sequence $\{p_n\}$ in W is defined by

$$\begin{cases} p_0 \in W \\ r_n = (1-\beta_n)p_n + \beta_n S p_n \\ q_n = T((1-\alpha_n)p_n + \alpha_n r_n) \\ p_{n+1} = S q_n. \end{cases} \tag{1.4}$$

where $\alpha_n, \beta_n \in [0,1]$
In 2018 Ullah et. al. [16] introduced a new three-step iteration process Known as the "M-Iteration Process", defined as let $S : W \to W$ be any nonlinear mapping. For each $p_0 \in W$, the sequence $\{p_n\}$ in W is defined by

$$\begin{cases} p_0 \in W \\ r_n = (1-\alpha_n)p_n + \alpha_n S p_n \\ q_n = S r_n \\ p_{n+1} = S q_n \end{cases} \tag{1.5}$$

where $\alpha_n \in [0,1]$
Ullah et al. [16] provided the weak and strong convergence of the M-iteration Scheme, for the class of Generalized nonexpansive mapping. In this paper, using the M -Iteration scheme with the class of mean nonexpansive mappings, and in this way, we extend his results in the more general setting of mean nonexpansive mappings.

2. Preliminaries

In this section we shall discuss some definitions and results to be used in the main results:

Definition 2.1. [6] A Banach space Y is called uniformly convex if for each $\epsilon \in (0,2]$ there is a $\delta > 0$ such that $p, q \in Y$

$$\begin{cases} ||p|| \leq 1, \\ ||q|| \leq 1, \\ ||p-q|| \geq \epsilon \end{cases} \Rightarrow ||\frac{p+q}{2}|| \geq \delta.$$

Definition 2.2. [12] A Banach space Y is said to satisfy the Opial's property if any weakly convergent sequence p_m in Y which admits a weak limit $c \in Y$, one has

$$\limsup_{m\to\infty} ||p_m - c|| < \limsup_{m\to\infty} ||p_m - c'||$$

for each $c' \in Y - \{c\}$.

Definition 2.3. [14] Suppose $W \neq \emptyset$ is any subset of a Banach space Y. A self map $S : W \to W$ is said to be endowed with the condition I if and only if a function $L : [0, \infty) \to [0, \infty)$ exists such that $L(0) = 0$ and $L(r) > 0$ for all $r > 0$ and $||p - Sp|| \geq L(d(p, F_s))$ for each $p \in W$.

In 1975 Zhang[20] introduced and studied the class of mean nonexpansive mappings in Banach spaces, he proved the unique existence of fixed points for this class of mappings in Banach spaces with normal structure. Zhang[20] provided the following class of mappings.

Definition 2.4. [20] Let $W \neq \emptyset$ be a subset of a Banach space. A self-map $S : W \to W$ is called mean nonexpansive if for all $p, q \in W$ there are non-negative real numbers a, b such that $a + b \leq 1$, we have

$$||Sp - Sq|| \leq a||p - q|| + b||p - Sq||.$$

Lemma 2.1. [13] *Let Y be a uniformly convex Banach space and $0 < u \leq t_n \leq v < 1 \; \forall n \in N$. Let $\{p_n\}$ and $\{q_n\}$ be two sequences of Y s.t. $\limsup_{n\to\infty} ||p_n|| \leq a$, $\limsup_{n\to\infty} ||q_n|| \leq a$ and $\limsup_{n\to\infty} ||t_n p_n + (1 - t_n)q_n|| = a$ hold for some $a \geq 0$. Then $\lim\limits_{n\to\infty} ||p_n - q_n|| = 0$.*

Lemma 2.2. [2] *If S is a self map and mean nonexpansive on a subset $W \neq \emptyset$ of a Banach Space Y. Then $F(S)$ is closed, Moreover, if Y is strictly convex and W is convex, then $F(S)$ is also convex.*

Lemma 2.3. [21] *Let S be a mean nonexpansive mapping of the Banach space Y. If S is continuous and $a + b < 1$, then S has a unique fixed point.*

Theorem 2.1. [21] *Let W be a nonempty closed subset of Banach space Y and S a mean nonexpansive self-mapping on W. If $a + 2b \leq 1$ and $b > 0$, then S has a unique fixed point.*

Theorem 2.2. [21] *Let Y be a real reflexive Banach space which satisfies Opial's condition, W a nonempty bounded closed convex subset of Y, and $S : W \to W$ a mean nonexpansive. Then S has a fixed point.*

Theorem 2.3. [21] *Let $W \neq \emptyset$ be a subset of a reflexive Banach space Y having Opial property. Let $S : W \to W$ be a mean nonexpansive mapping. If $\{p_n\} \subseteq W$ be such that*

(1) *$\{p_n\}$ converges weakly to e_0*
(2) $\lim_{n\to\infty} ||Sp_n - p_n|| = 0$

then $e_0 = Se_0$.

Corollary 2.1. [21] *Let W be a bounded closed convex subset of Banach space Y and S a mean nonexpansive self-mapping on W. Then S has an approximate fixed point sequence in W.*

3. Main results

In this section, we establish convergence theorems using M- iteration process (1.5). To prove this, the following Lemma are needed.

Lemma 3.4. *Let $W \neq \emptyset$ be a convex closed subset of a Uniformly Convex Banach Space Y and $S : W \to W$ be a mean nonexpansive mapping with $F(S) \neq \emptyset$. Assume that $\{p_n\}$ is a sequence defined by $M - iteration$ process (1.5). Then $\lim_{n\to\infty} ||p_n - e_0||$ exist for each $e_0 \in F(S)$.*

Proof. Let $e_0 \in F(S)$. Then using $M - iteration$ Process, we have

$$\begin{aligned}
||r_n - e_0|| &= ||(1-\alpha_n)p_n + \alpha_n Sp_n - e_0|| \\
&\leq (1-\alpha_n)||p_n - e_0|| + \alpha_n||Sp_n - e_0|| \\
&= (1-\alpha_n)||p_n - e_0|| + \alpha_n||Sp_n - Se_0|| \\
&\leq (1-\alpha_n)||p_n - e_0|| + \alpha_n(a||p_n - e_0|| + b||p_n - Se_0||) \\
&= (1-\alpha_n)||p_n - e_0|| + \alpha_n(a+b)||p_n - e_0|| \\
&\leq (1-\alpha_n)||p_n - e_0|| + \alpha_n||p_n - e_0|| \\
&= ||p_n - e_0|| \qquad (3.6)
\end{aligned}$$

and so

$$\begin{aligned}
||q_n - e_0|| &= ||Sr_n - e_0|| \\
&\leq a||r_n - e_0|| + b||r_n - Se_0|| \\
&= (a+b)||r_n - e_0|| \\
&\leq ||r_n - e_0|| \qquad (3.7)
\end{aligned}$$

This implies that

$$\begin{aligned}
||p_{n+1} - e_0|| &= ||Sq_n - e_0| \\
&\leq a||q_n - e_0|| + b||q_n - Se_0|| \\
&= (a+b)||q_n - e_0|| \\
&\leq ||q_n - e_0|| \qquad (3.8)
\end{aligned}$$

From equation (3.6), (3.7) and (3.8), we obtain

$$||p_{n+1} - e_0|| \leq ||p_n - e_0|| \qquad (3.9)$$

It follows that $\{||p_n - e_0||\}$ is non-increasing and bounded. Hence $\lim_{n\to\infty} ||p_n - e_0||$ exist. □

Lemma 3.5. *Let $W \neq \emptyset$ be a convex closed subset of a Uniformly Convex Banach Space Y and $S : W \to W$ be a mean nonexpansive mapping with $F(S) \neq \emptyset$. Assume that $\{p_n\}$ is a sequence defined by $M - iteration$ process (1.5). Consequently $\{p_n\}$ is bounded in Y with the property $\lim_{n\to\infty} ||Sp_n - p_n|| = 0$.*

Proof. Let $e_0 \in F(S)$. Using Lemma 3.4, we have $\lim_{n\to\infty} ||p_n - e_0||$ exist and $\{p_n\}$ is bounded. Suppose that

$$\lim_{n\to\infty} ||p_n - e_0|| = a$$

Case I: $a = 0$, we are done.
Case II: $a > 0$, From equation (3.6) in Lemma 3.4, we have

$$\begin{aligned}
&||r_n - e_0|| \leq ||p_n - e_0|| \\
\Rightarrow \limsup_{n\to\infty} ||r_n - e_0|| &\leq \limsup_{n\to\infty} ||p_n - e_0|| = a \qquad (3.10)
\end{aligned}$$

Now

$$\begin{aligned}
||Sp_n - e_0|| &= ||Sp_n - Se_0|| \\
&\leq a||p_n - e_0|| + b||p_n - Se_0|| \\
&= (a+b)||p_n - e_0|| \\
&\leq ||p_n - e_0|| \qquad (3.11)
\end{aligned}$$

It follows that

$$\limsup_{n\to\infty} ||Sp_n - e_0|| \leq \limsup_{n\to\infty} ||p_n - e_0|| = a$$

Using Lemma 3.4

$$||p_{n+1} - e_0|| \leq ||r_n - e_0||$$

It follows that

$$a \leq \liminf_{n\to\infty} ||r_n - e_0|| \tag{3.12}$$

From equation (3.10) and (3.12), we obtain

$$\begin{aligned} a &= \lim_{n\to\infty} ||r_n - e_0|| \\ &= \lim_{n\to\infty} ||(1-\alpha_n)p_n + \alpha_n Sp_n - e_0|| \\ &= \lim_{n\to\infty} ||(1-\alpha_n)(p_n - e_0) + \alpha_n(Sp_n - e_0)|| \end{aligned}$$

By Lemma 2.1

$$\lim_{n\to\infty} ||Sp_n - p_n|| = 0$$

□

Theorem 3.4. *Let $W \neq \emptyset$ be a convex closed subset of a Uniformly Convex Banach Space Y and $S : W \to W$ be a mean nonexpansive mapping with $F(S) \neq \emptyset$. Assume that $\{p_n\}$ is a sequence defined by $M - iteration$ process (1.5). If Y has the Opial property, then $\{p_n\}$ converges weakly to a point of $F(S)$.*

Proof. By Lemma 3.5, the sequence $\{p_n\}$ is bounded and $\lim_{n\to\infty} ||Sp_n - p_n|| = 0$. Since Y is Uniformly Convex Banach Space, it follows that Y is Reflexive Banach space. Thus there exists a weakly convergent subsequence $\{p_{n_i}\}$ of $\{p_n\}$ with some weak limit $q_1 \in W$. By Theorem 2.3, $q_1 \in F(S)$. Next we have to show that $\{p_n\}$is weakly convergent to q_1. We may suppose that $\{p_n\}$ is not weakly convergent to q_1, that is, there exists a weakly convergent subsequence $\{p_{n_j}\}$ of $\{p_n\}$ with a weak limit $q_2 \in W$ and $q_2 \neq q_1$. Again applying Theorem 2.3, $\{q_2\} \in F(S)$. By applying Opial's condition and using Lemma 3.4, it follows that

$$\begin{aligned} \lim_{n\to\infty} ||p_n - q_1|| &= \lim_{i\to\infty} ||p_{n_i} - q_1|| \\ &< \lim_{i\to\infty} ||p_{n_i} - q_2|| \\ &= \lim_{n\to\infty} ||p_n - q_2|| \\ &= \lim_{j\to\infty} ||p_{n_j} - q_2|| \\ &< \lim_{j\to\infty} ||p_{n_j} - q_1|| \\ &= \lim_{n\to\infty} ||p_n - q_1||. \end{aligned}$$

Which is contradiction, so $q_1 = q_2$. Thus $\{p_n\}$ converges weakly to $q_1 \in F(S)$. □

Theorem 3.5. *Let $W \neq \emptyset$ be a convex closed subset of a Uniformly Convex Banach Space Y and $S : W \to W$ be a mean nonexpansive mapping with $F(S) \neq \emptyset$. Assume that $\{p_n\}$ is a sequence defined by $M - iteration$ process (1.5). If W is compact, then $\{p_n\}$ converges strongly to an element of $F(S)$.*

Proof. Since W is compact and $\{p_n\} \subseteq W$. So we can choose a strongly convergent subsequence $\{p_{n_k}\}$ of $\{p_n\}$ such that $p_{n_k} \to u$. Now we show that $Su = u$.
For this

$$\begin{aligned} ||u - Su|| &\le ||u - p_{n_k}|| + ||p_{n_k} - Sp_{n_k}|| + ||Sp_{n_k} - Su|| \\ &\le ||u - p_{n_k}|| + ||p_{n_k} - Sp_{n_k}|| + (a||p_{n_k} - u|| + b||u - Sp_{n_k}||) \\ &\le ||u - p_{n_k}|| + ||p_{n_k} - Sp_{n_k}|| + (a||p_{n_k} - u|| + b||u - p_{n_k}|| + b||p_{n_k} - Sp_{n_k}||) \\ &= (a + b + 1)||u - p_{n_k}|| + (b + 1)||p_{n_k} - Sp_{n_k}|| \end{aligned}$$

Consequently

$$||u - Su|| \le (a + b + 1)||u - p_{n_k}|| + (b + 1)||p_{n_k} - Sp_{n_k}|| \tag{3.13}$$

According to Lemma 3.5, $\lim_{k\to\infty} ||p_{n_k} - Sp_{n_k}|| = 0$, so taking $k \to \infty$, from (3.13) implies $Su = u$. This shows that $u \in F(S)$. By Lemma 3.4, $\lim_{n\to\infty} ||p_n - u||$ exists. Consequently, u is the strong limit of $\{p_n\}$ and element of $F(S)$. □

The strong convergence theorem without the compactness assumption is established as follows

Theorem 3.6. *Let $W \neq \emptyset$ be a convex closed subset of a Uniformly Convex Banach Space Y and $S : W \to W$ be a mean nonexpansive mapping with $F(S) \neq \emptyset$. Assume that $\{p_n\}$ is a sequence defined by $M - iteration\ process$ (1.5). Then $\{p_n\}$ converges strongly to an element of $F(S)$ if and only if* $\liminf_{n\to\infty} d(p_n, F(S)) = 0$, *, where $d(p, F(S)) = \inf\{||p - e_0|| : e_0 \in F(S)\}$.*

Proof. Let $\{p_n\}$ converges to e_0, then $\lim_{n\to\infty} d(p_n, e_0) = 0$. It follows that $\lim_{n\to\infty} d(p_n, F(S)) = 0$. Therefore, $\liminf_{n\to\infty} d(p_n, F(S)) = 0$.
Conversely: Suppose that $\liminf_{n\to\infty} d(p_n, F(S)) = 0$ and $e_0 \in F(S)$. From the Lemma 3.4, $\lim_{n\to\infty} ||p_n - e_0||$ exists. Therefore $\lim_{n\to\infty} d(p_n, F(S)) = 0$, by assumption. We prove that $\{p_n\}$ is a Cauchy sequence in W. As $\lim_{n\to\infty} d(p_n, F(S)) = 0$, for a given $\epsilon > 0$, there exists $r_0 \in N$ such that for each $m \ge r_0$.

$$d(p_n, F(S)) < \frac{\epsilon}{2} \Rightarrow inf\{||p_n - e_0|| : e_0 \in F(S)\} < \frac{\epsilon}{2} \tag{3.14}$$

In particular $inf\{||p_{r_0} - e_0|| : e_0 \in F_s\} < \frac{\epsilon}{2}$. Therefore there exists $e_0 \in F(S)$ such that

$$||p_{r_0} - e_0|| < \frac{\epsilon}{2} \tag{3.15}$$

Now for $r, n \ge k_0$,

$$\begin{aligned} ||p_{n+r} - p_n|| &\ge ||p_{n+r} - e_0|| + ||p_n - e_0|| \\ &\le ||p_{r_0} - e_0|| + ||p_{r_0} - e_0|| \\ &= 2||p_{r_0} - e_0|| < \epsilon \end{aligned} \tag{3.16}$$

This shows that $\{p_n\}$ is a Cauchy sequence in W. As W is closed subset of a Banach Space Y, so there exists a point $e_0 \in W$ such that $\lim_{n\to\infty} p_n = e_0$. Now $\lim_{n\to\infty} d(p_n, F(S)) = 0$ gives that $d(e_0, F(S)) = 0$. Since From Lemma 2.2, we have have the set $F(S)$ a closed set in W. Hence $e_0 \in F(S)$. □

Theorem 3.7. *Let $W \neq \emptyset$ be a convex closed subset of a Uniformly Convex Banach Space Y and $S : W \to W$ be a mean nonexpansive mapping with $F(S) \neq \emptyset$. Assume that $\{p_n\}$ is a sequence defined by $M - iteration\ process$ (1.5). If S is endowed with condition (I), then $\{p_n\}$ converges strongly to an element of $F(S)$.*

Proof. From Lemma 3.5, we have

$$\lim_{n\to\infty} ||Sp_n - p_n|| = 0. \tag{3.17}$$

From Condition (I) and equation (3.17), we have

$$\lim_{n\to\infty} L(d(p_n, F(S))) \leq \lim_{n\to\infty} ||p_n - Sp_n||$$
$$\Rightarrow \lim_{n\to\infty} L(d(p_n, F(S))) = 0.$$

Since $L : [0,\infty) \to [0,\infty)$ is a non decreasing function satisfying $L(0) = 0$ and $L(r) > 0$ for all $r \in (0,\infty)$, therefore, we have

$$\lim_{n\to\infty} d(p_n, F(S)) = 0.$$

By Theorem 3.6, the sequence $\{p_n\}$ strongly converges to a fixed point of $F(S)$. □

4. Example

In this section, we discuss the examples of a mean nonexpansive mapping. Here it is illustrated that a mean nonexpansive mapping is not necessarily nonexpansive.

Example 4.1. [5] Let $S : [0,1] \to [0,1]$ be a mapping defined by

$$S(p) = \begin{cases} \frac{1}{4}, & p \in [0, \frac{1}{2}] \\ \frac{1}{8}, & p \in (\frac{1}{2}, 1) \\ 0, & p = 1 \end{cases}$$

Then S is mean nonexpansive with $a = b = \frac{1}{2}$, but S is not continuous. Therefore S cannot be a nonexpansive mapping.

Example 4.2. Let $S : [0,1] \to [0,1]$ be a mapping defined by

$$S(p) = \begin{cases} \frac{p}{7}, & p \in [0, \frac{1}{2}) \\ \frac{p}{8}, & p \in [\frac{1}{2}, 1] \end{cases}$$

Here S is discontinuous at $p = \frac{1}{2}$; consequently S is not nonexpansive mapping. Now we prove that S is mean nonexpansive mapping.
Case I: If $p, q \in [0, \frac{1}{2})$, By definition of S,

$$\begin{aligned} ||Sp - Sq|| &= ||\frac{p}{7} - \frac{q}{7}|| \\ &= \frac{1}{6}||\frac{6p}{7} - \frac{6q}{7}|| \\ &= \frac{1}{6}||p - \frac{q}{7} + \frac{q}{7} - \frac{p}{7} - (q - p + p - \frac{q}{7})|| \\ &\leq \frac{1}{6}||p - q|| + \frac{1}{3}||p - Sq|| + \frac{1}{6}||Sp - Sq|| \end{aligned}$$

This Implies that $||Sp - Sq|| \leq \frac{1}{5}||p-q|| + \frac{2}{5}||p - Sq||$.

Case II : If $p \in [0, \frac{1}{2})$ and $q \in [\frac{1}{2}, 1]$, By definition of S,

$$\begin{aligned}||Sp - Sq|| &= ||\frac{p}{7} - \frac{q}{8}|| \\ &= ||\frac{p}{7} - \frac{Sp}{7} + \frac{Sp}{7} - \frac{Sq}{7} + \frac{Sq}{7} - \frac{q}{8}|| \\ &\leq \frac{1}{7}||p - Sp|| + \frac{1}{7}||Sp - Sq|| + \frac{1}{7}||q - Sq|| \\ &\leq \frac{1}{7}||p - q|| + \frac{2}{7}||p - Sq|| + \frac{2}{7}||Sp - Sq||\end{aligned}$$

This Implies that $||Sp - Sq|| \leq \frac{1}{5}||p-q|| + \frac{2}{5}||p - Sq||$.

Case III: If $p \in [\frac{1}{2}, 1]$ and $q \in [0, \frac{1}{2})$, The proof is the same as in Case II.

Case IV: If $p, q \in [\frac{1}{2}, 1]$, The proof is the same as in Case I.

Hence, S is mean nonexpansive by taking $a = \frac{1}{5}$,$b = \frac{2}{5}$.

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