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Decomposition Of Intuitionistic Fuzzy Primary Ideals Of Γ-Rings

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ABSTRACT. In this paper, we establish the intuitionistic fuzzy version of the Lasker-Noether theorem for a commutative Γ-ring. We show that in a commutative Noetherian Γ-ring, every intuitionistic fuzzy ideal A can be decomposed as the intersection of a finite number of intuitionistic fuzzy irreducible ideals (primary ideals). This decomposition is called an intuitionistic fuzzy primary decomposition. Further, we show that in case of a minimal intuitionistic fuzzy primary decomposition of A, the set of all intuitionistic fuzzy associated prime ideals of A is independent of the particular decomposition. We also discuss some other fundamental results pertaining to this concept.

1. INTRODUCTION

The concept of a Γ-ring has a special place among generalizations of rings. One of the most interesting examples of a ring would be the endomorphism ring of an abelian group, i.e., $EndM$ or $Hom(M, M)$ where M is an abelian group. Now if two abelian groups, say A and B instead of one are taken, then $Hom(A, B)$ is no longer a ring in the way as $EndM$ becomes a ring because the composition is no longer defined. However, if one takes an element of $Hom(B, A)$ and put it in between two elements of $Hom(A, B)$, then the composition can be defined. This served as a motivating factor for studying the notion of Γ-ring. Another reason for studying Γ-ring is that the set M of rectangular matrices of the type $m \times n (m \neq n)$ over a division ring is closed under the composition of addition matrices but not under the composition of multiplication matrices. However multiplication can be performed by inserting a rectangular matrix of the type $n \times m$ from the set Γ in between the two elements from the set M. These structures provide a suitable setting for the study of rectangular matrices. The notion of a Γ-ring, a generalization of the concept of associative rings, has been introduced and studied by Nobusawa in [9]. Barnes [3] slightly weakened the conditions in the definition of a Γ-ring in the sense of Nobusawa. The structure of Γ-rings was investigated by several authors such as Barnes in [3], Kyuno in [7, 8]. Warsi in [18] studied the decomposition of primary ideal of Γ-rings. Paul in [10] studied various types of ideals of Γ-rings and the corresponding operator rings.

The concept of an intuitionistic fuzzy set as a generalization of Zadeh$'$s fuzzy sets has been proposed by Atanassov [1, 2]. Based on the intuitionistic fuzzy sets concept, Kim, Jun and Ozturk developed the notion of intuitionistic fuzzy ideals Γ-ring in [6]. Palaniappan et al. in [11, 12] studied in detail the properties of intuitionistic fuzzy ideals of Γ-rings. The concept of intuitionistic fuzzy prime ideal in Γ-ring was also innovated by Palaniappan and Ramachandran in [13, 14]. Sharma and Lata in [15] innovated the study of intuitionistic fuzzy characteristic ideals of Γ-ring and its operator ring. Sharma et al.

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introduced and studied the concepts of intuitionistic fuzzy prime radical and intuitionistic fuzzy primary ideal of Γ-ring in [16]. They also investigated the topological structure space on the set of all intuitionistic fuzzy prime ideals of Γ-ring in [17].

The decomposition of an ideal in terms of primary ideals is a traditional pillar of ideal theory. It provides the algebraic foundation for decomposing an algebraic variety into its irreducible components. From another point of view, a primary decomposition provides a generalisation of the factorization of an integer as a product of prime powers. An ideal I of a ring M has a primary decomposition, if $I = \bigcap_{i=1}^{k} Q_i$, where each Q_i is a primary ideal in M. Further, if no $Q_j \supset \bigcap_{i=1, j\neq i}^{n} Q_i, \forall j, 1 \leq j \leq k$ and the prime ideals $P_i = \sqrt{Q_i}$ are all distinct, then the primary decomposition is termed as minimal and the set $Ass(I) = \{P_1, P_2, ..., P_k\}$ is termed as the set of associated prime ideals of I. (For detail see [4, 5]).

In this paper, we study the intuitionistic fuzzy primary decomposition and minimal intuitionistic fuzzy primary decomposition of an intuitionistic fuzzy ideal in the Noetherian Γ-ring.

2. Preliminaries

Let us recall some definitions and results, which are necessary for the development of the paper,

Definition 2.1. ([3, 8, 9, 18]) If $(M, +)$ and $(\Gamma, +)$ are additive Abelian groups. Then M is called a **Γ-ring** (in the sense of Barnes [3]) if there exist mapping $M \times \Gamma \times M \to M$ [image of (x, α, y) is denoted by $x\alpha y$, $x, y \in M, \gamma \in \Gamma$] satisfying the following conditions:
(1) $x\alpha y \in M$.
(2) $(x + y)\alpha z = x\alpha z + y\alpha z$, $x(\alpha + \beta)y = x\alpha y + x\beta y$, $x\alpha(y + z) = x\alpha y + x\alpha z$.
(3) $(x\alpha y)\beta z = x\alpha(y\beta z)$. for all $x, y, z \in M$, and $\gamma \in \Gamma$.

The Γ-ring M is called **commutative** if $x\gamma y = y\gamma x, \forall x, y \in M, \gamma \in \Gamma$. An element $1 \in M$ is said to be the **unity** of M, if for each $x \in M$ there exists $\gamma \in \Gamma$ such that $x\gamma 1 = 1\gamma x = x$. A subset I of a Γ-ring M is a **left (right) ideal** of M if I is an additive subgroup of M and $M\Gamma I = \{x\alpha y | x \in M, \alpha \in \Gamma, y \in I\}$, $(I\Gamma M)$ is contained in I. If I is both a left and a right ideal, then I is a two-sided ideal, or simply an **ideal** of M. An ideal I of a Γ-ring M is called **prime (primary)** if for any ideals U, V of M such that $U\Gamma V \subseteq I$ implies $U \subseteq I$ or $V \subseteq I$ ($U \subseteq I$ or $V \subseteq \sqrt{I}$), where $\sqrt{I} = \{x \in M : (x\gamma)^{n-1}x \in I$ for some $n \in \mathbf{N}$ and $\gamma \in \Gamma\}$ is the **radical ideal** of I, here for $n = 1$, $(x\gamma)^{n-1}x = x$ ([18]).

We now review some intuitionistic fuzzy logic concepts. We refer the reader to follow [1, 2] and [16, 17] for complete details.

Definition 2.2. ([1, 2]) An **intuitionistic fuzzy set** A in X can be represented as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) >: x \in X\}$, where the functions $\mu_A, \nu_A : X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Remark 2.1. ([1, 2])
(i)When $\mu_A(x) + \nu_A(x) = 1$, i.e., $\nu_A(x) = 1 - \mu_A(x) = \mu_{A^c}(x)$, then A is called a **fuzzy set**.
(ii) We write by $IFS(X)$, the set of all intuitionistic fuzzy sets in X.

If $A, B \in IFS(X)$, then $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x), \forall x \in X$. For any subset Y of X, the **intuitionistic fuzzy characteristic function** χ_Y is an intuitionistic fuzzy set of X, defined as $\chi_Y(x) = (1,0), \forall x \in Y$ and $\chi_Y(x) = (0,1), \forall x \in X \backslash Y$. Let $p, q \in [0,1]$ with $p + q \leq 1$. Then the crisp set $A_{(p,q)} = \{x \in X : \mu_A(x) \geq p$ and $\nu_A(x) \leq q\}$ is called the (p,q)-**level cut subset** of A. We denote $A_{(1,0)}$ by A_*. Also the IFS $x_{(p,q)}$ of X is defined as $x_{(p,q)}(y) = (p,q)$, if $y = x$, otherwise $(0,1)$ is called the **intuitionistic fuzzy point (IFP)** in X with **support** x. By $x_{(p,q)} \in A$ we mean $\mu_A(x) \geq p$ and $\nu_A(x) \leq q$. Thus $x_{(p,q)} \in A$ if and only if $x_{(p,q)} \subseteq A$.

Definition 2.3. ([6, 11, 16]) Let A be an IFS of a Γ-ring M. Then A is called an **intuitionistic fuzzy ideal** (IFI) of M if for all $m, n \in M, \gamma \in \Gamma$, the following are satisfied
(i) $\mu_A(m - n) \geq \mu_A(m) \wedge \mu_A(n)$;
(ii) $\mu_A(m\gamma n) \geq \mu_A(m) \vee \mu_A(n)$;
(iii) $\nu_A(m - n) \leq \nu_A(m) \vee \nu_A(n)$;
(iv)$\nu_A(m\gamma n) \leq \nu_A(m) \wedge \nu_A(n)$.

Definition 2.4. ([6, 11, 16]) Let A, B be two IFSs of a Γ-ring M. Then the **product** $A\Gamma B$ of A and B is defined by

$$(\mu_{A\Gamma B}(x), \nu_{A\Gamma B}(x)) = \begin{cases} (Sup_{x=y\gamma z}\{\mu_A(y) \wedge \mu_B(z)\}, Inf_{x=y\gamma z}\{\nu_A(y) \vee \nu_B(z)\}), & \text{if } x = y\gamma z \\ (0,1), & \text{otherwise} \end{cases}$$

Definition 2.5. ([13, 16, 17]) Let M be a Γ-ring. A non-constant IFI P of M is called an **intuitionistic fuzzy prime ideal (IFPI)** of M, if for all pair of IFIs A, B of M, $A\Gamma B \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 2.6. ([16, 17]) Let M be a Γ-ring. For any IFI A of M. The IFS $\sqrt{A}$ defined by

$$\mu_{\sqrt{A}}(x) = \vee\{\mu_A((x\gamma)^{n-1}x) : n \in \mathbf{N}\} \text{ and } \nu_{\sqrt{A}}(x) = \wedge\{\nu_A((x\gamma)^{n-1}x) : n \in \mathbf{N})\}$$

is called the **intuitionistic fuzzy prime radical** of A, where $(x\gamma)^{n-1}x = x$, for $n = 1, \gamma \in \Gamma$.

Proposition 2.1. ([16, 17]) *For every IFIs A and B of* Γ*-ring* M*, we have*
(i) $A \subseteq \sqrt{A}$*;*
(ii) $A \subseteq B \Rightarrow \sqrt{A} \subseteq \sqrt{B}$*;*
(iii) $\sqrt{\sqrt{A}} = \sqrt{A}$*.*

Proposition 2.2. ([16]) *Let* A, B *be two IFIs of a* Γ*-ring* M*. Then*

$$\sqrt{A\Gamma B} = \sqrt{A \cap B} = \sqrt{A} \cap \sqrt{B}.$$

Definition 2.7. ([16]) *Let* Q *be a non-constant IFI of a* Γ*-ring* M*. Then* Q *is said to be an* ***intuitionistic fuzzy primary ideal*** *of* M *if for any two IFIs* A, B *of* M *such that* $A\Gamma B \subseteq Q$ *implies that either* $A \subseteq Q$ *or* $B \subseteq \sqrt{Q}$*.*

Theorem 2.1. ([16]) *Let* M *be a commutative* Γ*-ring and* Q *be an IFI of* M*. Then for any two IFPs* $x_{(p,q)}, y_{(t,s)} \in IFP(M)$ *the following are equivalent:*
(i) Q *is an intuitionistic fuzzy prime (primary) ideal of* M
(ii) $x_{(p,q)}\Gamma y_{(t,s)} \subseteq Q$ *implies* $x_{(p,q)} \subseteq Q$ *or* $y_{(t,s)} \subseteq Q$ *(* $x_{(p,q)} \subseteq Q$ *or* $y_{(t,s)} \subseteq \sqrt{Q}$*).*

Theorem 2.2. ([16]) *If* Q *is an intuitionistic fuzzy prime (primary) ideal of a* Γ*-ring* M*, then the following conditions hold:*
(i) $Q(0_M) = (1,0)$*,*
(ii) Q_* *is a prime (primary) ideal of* M*,*
(iii) $Img(Q) = \{(1,0),(t,s)\}$*, where* $t, s \in [0,1)$ *such that* $t + s \leq 1$*.*

Definition 2.8. ([16]) Let Q be an intuitionistic fuzzy primary ideal of Γ-ring M and $P = \sqrt{Q}$ which is an intuitionistic fuzzy prime ideal of M. Then Q is called an **intuitionistic fuzzy P-primary ideal** of M.

Theorem 2.3. ([14]) *A Γ-ring M is Noetherian if and only if the set of values of any IFI of M is well-ordered subset of* $[0, 1]$.

Theorem 2.4. ([14]) *Let every decreasing chain of ideals terminates at finite step in Γ-ring M. For an IFI A of M, A has finite number of intuitionistic values, that is, μ_A and ν_A have finite number of values.*

Theorem 2.5. ([14]) *Let A be an IFS in a Γ-ring M and $Im(A) = \{(\alpha_0, \beta_0), (\alpha_1, \beta_1), ..., (\alpha_k, \beta_k)\}$ with $(\alpha_i, \beta_i) < (\alpha_j, \beta_j)$ whenever $i > j$. Let $\{U_n | n = 0, 1, ..., k\}$ be the family of ideals of M such that*

(1) $U_0 \subset U_1 \subset .... \subset U_{k-1} \subset U_k = M$
(2) $A(U_n^*) = (\alpha_n, \beta_n)$,

that is $\mu_A(U_n^*) = \alpha_n$, $\nu_A(U_n^*) = \beta_n$, where $U_n^* = U_n \setminus U_{n-1}$, $U_{-1} = \emptyset$, for $n = 0, 1, ..., k$. Then A is an IFI of M.

Remark 2.2. ([4]) In a commutative Noetherian ring, the notion of an irreducible ideal can be used to prove the Lasker-Noether theorem: every ideal (in a Noetherian ring) has a primary decomposition.

3. Intuitionistic fuzzy irreducibile ideals

In this section, we study the irreducibility of an intuitionistic fuzzy ideal and prove some relations between intuitionistic fuzzy prime ideals, intuitionistic fuzzy irreducible ideals and intuitionistic fuzzy primary ideals. We first prove that every intuitionistic fuzzy ideal in a Noetherian ring can be written as a finite intersection of intuitionistic fuzzy irreducible ideals, where the intuitionistic fuzzy ideal takes only two values.

Definition 3.9. Let A be an intuitionistic fuzzy ideal of a Γ-ring M. We say that A is an **intuitionistic fuzzy irreducible** if A cannot be expressed as the intersection of two intuitionistic fuzzy ideals of M properly containing A; otherwise A is called **reducible**.

Thus A is an intuitionistic fuzzy irreducible ideal if and only if, whenever $A = A_1 \cap A_2$ with A_1, A_2 intuitionistic fuzzy ideals of M, then either $A = A_1$ or $A = A_2$.

Proposition 3.3. *Let A be a non-constant IFI of a Γ-ring M. Then A is an intuitionistic fuzzy irreducible ideal of M if and only if the following hold:*

(1) A_* is an irreducible ideal of M
(2) $Im(A) = \{(1, 0), (\alpha, \beta)\}$, where $\alpha, \beta \in [0, 1)$ such that $\alpha + \beta \leq 1$.
(3) A is of the form

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A_* \\ \alpha, & \text{if } x \in M \setminus A_* \end{cases} ; \quad \nu_A(x) = \begin{cases} 0, & \text{if } x \in A_* \\ \beta, & \text{if } x \in M \setminus A_*. \end{cases}$$

Proof. Firstly suppose that A be an intuitionistic fuzzy irreducible ideal of M. Let $A_* = I \cap J$ for some ideals I, J of M. We have $A_* \subseteq I$ and $A_* \subseteq J$. If possible, let $A_* \neq I$ and $A_* \neq J$.
Then $(I \setminus A_*) \cap (J \setminus A_*)$ is empty. Let us define two IFSs A_1 and A_2 as follows:

$$\mu_{A_1}(x) = \begin{cases} 1, & \text{if } x \in A_* \\ t_1, & \text{if } x \in I \setminus A_* \\ t_2, & \text{if } x \in M \setminus I \end{cases} ; \quad \nu_{A_1}(x) = \begin{cases} 0, & \text{if } x \in A_* \\ s_1, & \text{if } x \in I \setminus A_* \\ s_2, & \text{if } x \in M \setminus I. \end{cases}$$

and

$$\mu_{A_2}(x) = \begin{cases} 1, & \text{if } x \in A_* \\ t_1, & \text{if } x \in J \setminus A_* \\ t_2, & \text{if } x \in M \setminus J \end{cases}; \quad \nu_{A_2}(x) = \begin{cases} 0, & \text{if } x \in A_* \\ s_1, & \text{if } x \in J \setminus A_* \\ s_2, & \text{if } x \in M \setminus J. \end{cases}$$

Now, it is a straight forward case study to verify that A_1 and A_2 are IFIs of M and $A = A_1 \cap A_2$. Though we have $A \neq A_1$ and $A \neq A_2$. This contradict the fact that A is an intuitionistic fuzzy irreducible ideal of M. Consequently, $A_* = I$ or $A_* = J$ and hence A_* is an irreducible ideal of M.

Next we show that $(1,0) \in Im(A)$. If possible, suppose that $(1,0) \notin Im(A)$. Then $\mu_A(0) < 1, \nu_A(0) > 0$. Let us define two IFSs A_3 and A_4 as follows:

$$\mu_{A_3}(x) = \begin{cases} 1, & \text{if } x \in A_* \\ \mu_A(0), & \text{if otherwise} \end{cases}; \quad \nu_{A_3}(x) = \begin{cases} 0, & \text{if } x \in A_* \\ \nu_A(0), & \text{if otherwise.} \end{cases}$$

and $A_4(x) = A(0)$, for all $x \in M$. It is easy to verify that A_3 and A_4 are IFIs of M such that $A = A_3 \cap A_4$. But $A \subset A_3$ and $A \subset A_4$. Thus we arrive at a contradiction since A is an intuitionistic fuzzy irreducible ideal of M. Consequently $(1,0) \in Im(A)$.
Further, to show that $|Im(A)| = 2$. It is sufficient to show that chain of the level-cut set ideals is given by $A_* \subseteq M$. If possible, let the chain of the level-cut set ideals be $A_* \subseteq A_{(t_1,s_1)} \subseteq M$, where $t_1, s_1 \in (0,1)$ with $t_1 + s_1 \leq 1$. Then A is given by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A_* \\ t_1, & \text{if } x \in A_{(t_1,s_1)} \setminus A_* \\ t_2, & \text{if } x \in M \setminus A_{(t_1,s_1)} \end{cases}; \quad \nu_A(x) = \begin{cases} 0, & \text{if } x \in A_* \\ s_1, & \text{if } x \in A_{(t_1,s_1)} \setminus A_* \\ s_2, & \text{if } x \in M \setminus A_{(t_1,s_1)}. \end{cases}$$

where $t_2 < t_1$ and $s_2 > s_1$. Let us construct two IFSs A_5 and A_6 as follows:

$$\mu_{A_5}(x) = \begin{cases} 1, & \text{if } x \in A_{(t_1,s_1)} \\ \mu_A(x), & \text{if } x \in M \setminus A_{(t_1,s_1)} \end{cases}; \quad \nu_{A_5}(x) = \begin{cases} 0, & \text{if } x \in A_{(t_1,s_1)} \\ \nu_A(x), & \text{if } x \in M \setminus A_{(t_1,s_1)}. \end{cases}$$

and

$$\mu_{A_6}(x) = \begin{cases} 1, & \text{if } x \in A_* \\ t_1, & \text{if } x \in A_{(t_1,s_1)} \setminus A_* \\ t_3, & \text{if } x \in M \setminus A_{(t_1,s_1)} \end{cases}; \quad \nu_{A_6}(x) = \begin{cases} 0, & \text{if } x \in A_* \\ s_1, & \text{if } x \in A_{(t_1,s_1)} \setminus A_* \\ s_3, & \text{if } x \in M \setminus A_{(t_1,s_1)}. \end{cases}$$

where $t_2 < t_3 < t_1$ and $s_2 > s_3 > s_1$. It is routine case study to check that A_5 and A_6 are IFIs of M and $A = A_5 \cap A_6$. But $A \subset A_5$ and $A \subset A_6$. It contradict the fact that A is an intuitionistic fuzzy irreducible ideal of M. Consequently the chain of level cut-set ideal is $A_* \subseteq M$ and hence A is given by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in A_* \\ t_1, & \text{if } x \in M \setminus A_* \end{cases}; \quad \nu_A(x) = \begin{cases} 0, & \text{if } x \in A_* \\ s_1, & \text{if } x \in M \setminus A_*. \end{cases}$$

Hence $|Im(A)| = 2$.

Conversely, let the conditions hold. Assume that A is not an intuitionistic fuzzy irreducible ideal of M. Suppose that $A = A_7 \cap A_8$ for some IFIs A_7, A_8 of M with $A \subset A_7$ and $A \subset A_8$. Then there exists $x, y \in M$ such that $\mu_A(x) < \mu_{A_7}(x), \nu_A(x) > \nu_{A_7}(x)$ and $\mu_A(y) < \mu_{A_8}(y), \nu_A(y) > \nu_{A_8}(y)$. It follows that $x, y \notin A_*$. Now, if $x = y$, then $\mu_A(x) < \mu_{A_7 \cap A_8}(x)$ and $\nu_A(x) > \nu_{A_7 \cap A_8}(x)$, i.e., $A \subset A_7 \cap A_8$, which is a contradiction.

So $x \neq y$ implies $A_* \subseteq \langle A_*, x\rangle$ and $A_* \subseteq \langle A_*, y\rangle$. Therefore $A_* \subseteq \langle A_*, x\rangle \cap \langle A_*, y\rangle$. Let $z \in \langle A_*, x\rangle \cap \langle A_*, y\rangle$, then $z = m + r_1\gamma_1 x = n + r_2\gamma_2 y$, for some $m, n \in A_*$, $r_1, r_2 \in M$, $\gamma_1, \gamma_2 \in \Gamma$.

Therefore, $\mu_A(m-n) = \mu_A(-r_1\gamma_1 x + r_2\gamma_2 y) = 1$ and $\nu_A(m-n) = \nu_A(-r_1\gamma_1 x + r_2\gamma_2 y) = 0$ implies that $\mu_{A_7}(-r_1\gamma_1 x + r_2\gamma_2 y) = \mu_{A_8}(-r_1\gamma_1 x + r_2\gamma_2 y) = 1$ and $\nu_{A_7}(-r_1\gamma_1 x + r_2\gamma_2 y) = \nu_{A_8}(-r_1\gamma_1 x + r_2\gamma_2 y) = 0$. This imply $\mu_{A_7}(r_1\gamma_1 x) = \mu_{A_7}(r_2\gamma_2 y)$, $\nu_{A_7}(r_1\gamma_1 x) = \nu_{A_7}(r_2\gamma_2 y)$ and $\mu_{A_8}(r_1\gamma_1 x) = \mu_{A_8}(r_2\gamma_2 y)$, $\nu_{A_8}(r_1\gamma_1 x) = \nu_{A_8}(r_2\gamma_2 y)$.

But $\mu_{A_7}(r_1\gamma_1 x) \geq \mu_{A_7}(r_1) \vee \mu_{A_7}(x) \geq \mu_{A_7}(x) > \mu_A(x) = \alpha$.
Similarly $\nu_{A_7}(r_1\gamma_1 x) \leq \nu_{A_7}(r_1) \wedge \nu_{A_7}(x) \leq \nu_{A_7}(x) < \nu_A(x) = \beta$. This gives $r_1\gamma_1 x, r_2\gamma_2 y \in A_*$. Hence $z \in A_*$. Thus, we have $A_* = \langle A_*, x\rangle \cap \langle A_*, y\rangle$ with $A_* \subset \langle A_*, x\rangle$ and $A_* \subset \langle A_*, y\rangle$. This implies that A_* is not an irreducible ideal of M, which is a contradiction. □

Corollary 3.1. *Let I be an ideal of Γ-ring M. Then I is an irreducible ideal if and only if χ_I is an intuitionistic fuzzy irreducible ideal of M.*

Corollary 3.2. *If A be an intuitionistic fuzzy prime ideal of Γ-ring M. Then A is an intuitionistic fuzzy irreducible ideal of M.*

Proof. By Theorem (2.2) and Proposition (3.3) and the fact that every prime ideal in Γ-ring is an irreducible ideal. □

Note that the converse of Corollary (3.2) may not be true. See the following example:

Example 3.1. Consider $M = \Gamma = \mathbb{Z}$ be the additive group of integers. Then M is a Γ-ring. Consider the IFI A of M defined by

$$\mu_A(x) = \begin{cases} 1, & \text{if } x \in \langle 4\rangle \\ 0.4, & \text{if otherwise} \end{cases}; \quad \nu_A(x) = \begin{cases} 0, & \text{if } x \in \langle 4\rangle \\ 0.3, & \text{if otherwise.} \end{cases}$$

It is easy to check that A is an intuitionistic fuzzy irreducible ideal of M, but it is not an intuitioniatic fuzzy prime ideal of M, as $A_* = \langle 4\rangle$ is not a prime ideal in M.

Corollary 3.3. *If A be an intuitionistic fuzzy irreducible ideal of a Noetherian ring Γ-ring M, then A is an intuitionistic fuzzy primary ideal in M.*

Proof. From [[18], Lemma(4.2)] we see that every irreducible ideal in a Noetherian Γ-ring is a primary ideal. Then the result follows by Proposition (3.3) and Theorem (2.2). □

Proposition 3.4. *Let A be an intuitionsitic fuzzy ideal of a Noetherian Γ-ring M with $Im(A) = \{(1,0), (\alpha, \beta)\}$, where $\alpha, \beta \in [0,1)$ such that $\alpha + \beta \leq 1$. Then A can be expressed as a finite intersection of intuitionistic fuzzy irreducible ideals of M.*

Proof. By [[18], Lemma(4.1)], every ideal in a Noetherian Γ-ring is a finite intersection of irreducible ideals. Therefore, suppose that $A_* = \bigcap_{i=1}^{n} J_i$, J_i be an irreducible ideal of M. Define the IFIs $A_1, A_2, ..., A_n$ by

$$\mu_{A_i}(x) = \begin{cases} 1, & \text{if } x \in J_i \\ \alpha, & \text{if } x \notin J_i \end{cases}; \quad \nu_{A_i}(x) = \begin{cases} 0, & \text{if } x \in J_i \\ \beta, & \text{if } x \notin J_i. \end{cases}$$

Where $\alpha, \beta \in [0,1)$ such that $\alpha + \beta \leq 1$. Then by Proposition (3.3), for each $i = 1, 2, ..., n$, A_i is an intuitionisic fuzzy irreducible ideal of M and it is easy to check that $A = \bigcap_{i=1}^{n} A_i$. □

Proposition 3.5. *Let A be an intuitionsitic fuzzy ideal of a Noetherian Γ-ring M with $Im(A) = \{(1,0), (\alpha, \beta)\}$, where $\alpha, \beta \in [0,1)$ such that $\alpha + \beta \leq 1$. Then A can be expressed as a finite intersection of intuitionistic fuzzy primary ideals of M.*

Proof. This follows from Proposition (3.4) and Corollary (3.3) □

4. Decomposition of intuitionistic Fuzzy Primary Ideal of Γ-Ring

In this section, we study the decomposability of an intuitionistic fuzzy ideal in a Noetherian Γ-ring, in terms of intuitionistic fuzzy primary ideals that the set of their respective intuitionistic fuzzy radical ideals are independent of the particular decomposition.

To begin this section, we first recall the definition of residual quotient $(A : B)$ of an intuitionistic fuzzy ideal A by an intuitionistic fuzzy set B in a Γ-ring M.

Definition 4.10. For any IFI A of a Γ-ring M and for any IFS B of M, the **IF residual quotient** of A by B is denoted by $(A : B)$ and is defined as

$$(A : B) = \bigcup\{x_{(p,q)} \in IFP(M) : x_{(p,q)}\Gamma B \subseteq A\}$$

For any intuitionistic fuzzy point $x_{(p,q)}$ of Γ-ring M, we use a streamlined notation $(A : x_{(p,q)})$ for $(A : \langle x_{(p,q)}\rangle)$, where $\langle x_{(p,q)}\rangle = \bigcap\{C : C$ is an IFI of M such that $x_{(p,q)} \subseteq C\}$, be an IFI generated by $x_{(p,q)}$. There is no difficulty in seeing that $(A : x_{(p,q)})$ is an IFI of M and $A \subseteq (A : x_{(p,q)})$.

Proposition 4.6. *Let Q be an intuitioistic fuzzy P-primary ideal of Γ-ring M, where $P = \sqrt{Q}$. If $x_{(p,q)} \in IFP(M)$ be any intuitionistic fuzzy point of M. Then*
(i) If $x_{(p,q)} \in Q$, then $(Q : x_{(p,q)}) = \chi_M$;
(ii) If $x_{(p,q)} \notin Q$, then $(Q : x_{(p,q)})$ is an intuitionistic fuzzy P-primary ideal and $\sqrt{(Q : x_{(p,q)})} = P$;
(iii) If $x_{(p,q)} \notin \sqrt{Q}$, then $(Q : x_{(p,q)}) = Q$.

Proof. Let $x_{(p,q)} \in IFP(M)$, Q be an intuitionistic fuzzy primary ideal of M such that $P = \sqrt{Q}$.

(i) If $x_{(p,q)} \in Q$, then $(Q : x_{(p,q)}) = \bigcup\{y_{(t,s)} \in IFP(M) : y_{(t,s)}\Gamma x_{(p,q)} \subseteq Q\}$.
Now $(Q : x_{(p,q)}) \subseteq \chi_M$ always. For other inclusion.
Let $y_{(t,s)} \in \chi_M$ then $y_{(t,s)}\Gamma x_{(p,q)} = (y\Gamma x)_{(t\wedge p, s\vee q)} \subseteq Q$. This implies $y_{(t,s)} \in (Q : x_{(p,q)})$. Thus $\chi_M \subseteq (Q : x_{(p,q)})$. Hence $(Q : x_{(p,q)}) = \chi_M$.

(ii) Obviously $Q \subseteq (Q : x_{(p,q)})$. Let $y_{(t,s)} \in (Q : x_{(p,q)})$. So $y_{(t,s)}\Gamma x_{(p,q)} \subseteq Q$. Since $x_{(p,q)} \notin Q$ imply that $y_{(t,s)} \in \sqrt{Q} = P$. This means that $Q \subseteq (Q : x_{(p,q)}) \subseteq P$ and so $\sqrt{Q} \subseteq \sqrt{(Q : x_{(p,q)})} \subseteq \sqrt{P} = P$. This imply that $\sqrt{(Q : x_{(p,q)})} = P$.

Now we show that $(Q : x_{(p,q)})$ is an intuitionistic fuzzy primary ideal of M. Assume that for any $\gamma_1 \in \Gamma$ such that $a_{(u_1,v_1)}\gamma_1 b_{(u_2,v_2)} \in (Q : x_{(p,q)})$ and $b_{(u_2,v_2)} \notin \sqrt{(Q : x_{(p,q)})}$, then $a_{(u_1,v_1)}\gamma_1 b_{(u_2,v_2)}\gamma_2 x_{(p,q)} \in Q$, i.e., $(a_{(u_1,v_1)}\gamma_1 x_{(p,q)})\gamma_2 b_{(u_2,v_2)} \in Q$ and Q is intuitionistic fuzzy P-primary ideal of M, This implies that either $a_{(u_1,v_1)}\gamma_1 x_{(p,q)} \in Q$ or $b_{(u_2,v_2)} \in \sqrt{Q} = P = \sqrt{(Q : x_{(p,q)})}$. This imply $a_{(u_1,v_1)}\gamma_1 x_{(p,q)} \in Q$. Thus $a_{(u_1,v_1)} \in (Q : x_{(p,q)})$. Hence $(Q : x_{(p,q)})$ is an intuitionistic fuzzy primary ideal of M.

(iii) Since $Q \supseteq x_{(p,q)} \cap Q \supseteq x_{(p,q)}\Gamma Q$, i.e., $x_{(p,q)}\Gamma Q \subseteq Q$. Therefore by the properties of residual quotient, we have $Q \subseteq (Q : x_{(p,q)})$. Further, $x_{(p,q)}\Gamma(Q : x_{(p,q)}) \subseteq Q$. As Q is an intuitionistic fuzzy primary ideal of M and $x_{(p,q)} \notin \sqrt{Q}$ implies that $(Q : x_{(p,q)}) \subseteq Q$. Hence $(Q : x_{(p,q)}) = Q$. □

Proposition 4.7. *If $Q_1, Q_2, \ldots\ldots, Q_n$ be IFIs of Γ-ring M and $x_{(p,q)} \in IFP(M)$, then*
$$(\bigcap_{i=1}^{n} Q_i : x_{(p,q)}) = \bigcap_{i=1}^{n}(Q_i : x_{(p,q)}).$$

Proof. Now $y_{(t,s)} \in (\bigcap_{i=1}^{n} Q_i : x_{(p,q)})$
$\Leftrightarrow y_{(t,s)}\Gamma x_{(p,q)} \subseteq \bigcap_{i=1}^{n} Q_i$
$\Leftrightarrow y_{(t,s)}\Gamma x_{(p,q)} \subseteq Q_i, \forall i = 1, 2, \ldots., n$
$\Leftrightarrow y_{(t,s)} \in (Q_i : x_{(p,q)}), \forall i = 1, 2, \ldots., n$
$\Leftrightarrow y_{(t,s)} \in \bigcap_{i=1}^{n}(Q_i : x_{(p,q)})$.
Hence $(\bigcap_{i=1}^{n} Q_i : x_{(p,q)}) = \bigcap_{i=1}^{n}(Q_i : x_{(p,q)})$. □

In the following example, we show that if Q_1, Q_2 are two intuitionistic fuzzy primary ideals of a Γ-ring M, then $Q_1 \cap Q_2$ need not to be an intuitionistic fuzzy primary ideal of M.

Example 4.2. Let $M = \Gamma = \mathbb{Z}$, be the additive group of integers. Then M is a Γ-ring. Let $I = 2\mathbb{Z}, J = 3\mathbb{Z}$. Clearly, I, J are primary (in fact prime) ideal in M. Define $Q_1 = \chi_I, Q_2 = \chi_J$. Then by Theorem (2.2), Q_1, Q_2 are intuitionistic fuzzy primary ideals of M. Also, $Q_1 \cap Q_2 = \chi_{I\cap J} = \chi_{6\mathbb{Z}}$, which is not an intuitioniatic fuzzy primary ideal of M, as $6\mathbb{Z}$ is not a primary ideal of $\mathbb{Z}$.

Theorem 4.6. *Let $Q_1, Q_2, \ldots\ldots, Q_n$ be intuitionistic fuzzy P-primary ideals of Γ-ring M with $P = \sqrt{Q_i}, \forall i = 1, 2, \ldots, n$, an intitionistic fuzzy prime ideal of M. Then $Q = \bigcap_{i=1}^{n} Q_i$ is an intuitionistic fuzzy P-primary ideal of M.*

Proof. Let $x_{(p,q)}, y_{(t,s)} \in IFP(M)$ be such that $x_{(p,q)}\Gamma y_{(t,s)} \subseteq Q = \bigcap_{i=1}^{n} Q_i$ and $x_{(p,q)} \notin Q$. Then $x_{(p,q)} \notin Q_j$, for some $j \in \{1, 2, \ldots.., n\}$ also $x_{(p,q)}\Gamma y_{(t,s)} \subseteq Q_j$, for all $j \in \{1, 2, \ldots.., n\}$. Since each Q_j is an intuitionistic fuzzy P-primary ideals of M, we have

$$y_{(t,s)} \in \sqrt{Q_j} = P = \bigcap_{i=1}^{n} \sqrt{Q_i} = \sqrt{\bigcap_{i=1}^{n} Q_i} = \sqrt{Q}.$$

Hence Q is an intuitionistic fuzzy P-primary ideals of M. □

Definition 4.11. A **primary decomposition of an intuitionistic fuzzy ideal** A in a Γ-ring M is an expression of A as a finite intersection of intuitionistic fuzzy primary ideals Q_i, say $A = \bigcap_{i=1}^{n} Q_i$.

Definition 4.12. An intuitionistic fuzzy primary decomposition of an intuitionistic fuzzy ideal $A = \bigcap_{i=1}^{n} Q_i$ of Γ-ring M is said to be *minimal* if:

(1) all intuitionistic fuzzy primary ideal Q_i have distinct $\sqrt{Q_i}$;
(2) $\bigcap_{j\neq i=1}^{n} Q_j \nsubseteq Q_i$.

Remark 4.3. If intuitionistic fuzzy primary decomposition $A = \bigcap_{i=1}^{n} Q_i$ is not minimal, that is if $\sqrt{Q_j} = \sqrt{Q_k} = P$ for $j \neq k$, then we may achieve (1) of Definition (4.12) by replacing Q_j and Q_k by $Q' = Q_j \cap Q_k$ which is an intuitionistic fuzzy P-primary ideal of M by Theorem (4.6). Repeating this process, we get will arrive at an intuitionistic fuzzy primary decomposition in which all $\sqrt{Q_i}$ are distinct. If $\bigcap_{j\neq i=1}^{n} Q_j \subseteq Q_i$, we may simply omit Q_i. Repeating this process, we will achieve (2) of Definition (4.12).

Lemma 4.1. *Let $A_1, A_2, \ldots., A_n$ be IFIs of Γ-ring M and let P be an intuitionistic fuzzy prime ideal of M. Then*

(1) If $\bigcap_{i=1}^{n} A_i \subseteq P$, then $A_i \subseteq P$ for some i;
(2) If $\bigcap_{i=1}^{n} A_i = P$, then $A_i = P$ for some i.

Proof. **(1)** Suppose $A_i \nsubseteq P$ for all i. Then $\exists^s$, $(x_i)_{(p_i,q_i)} \in A_i$ such that $(x_i)_{(p_i,q_i)} \notin P$ for $1 \leq i \leq n$. Therefore $(x_1)_{(p_1,q_1)}\Gamma(x_2)_{(p_2,q_2)}\Gamma.....\Gamma(x_n)_{(p_n,q_n)} \subseteq A_1\Gamma A_2\Gamma....\Gamma A_n \subseteq \bigcap_{i=1}^n A_i \subseteq P$. But, since P is an intuitionistic fuzzy prime ideal and $A_1\Gamma A_2\Gamma....\Gamma A_n \subseteq P$, then $A_i \subseteq P$ for some i.

(2) If $P = \bigcap_{i=1}^n A_i$, then $P \subseteq A_i$ for some i, and from part (1), $A_i \subseteq P$ for some i. Hence $P = A_i$, for some i. □

Definition 4.13. An intuitionistic fuzzy prime ideal P in a Γ-ring M is called an **intuitionistic fuzzy associated prime ideal** of an IFI A if $P = \sqrt{(A : x_{(p,q)})}$ for some $x_{(p,q)} \in IFP(M)$.

Moreover, for an IFI A of a Γ-ring M. We define $IF - ASS(A)$ to be the set of all intuitionistic fuzzy prime ideals associated with the IFI A, i.e.,

$$IF - ASS(A) = \{\sqrt{(A : x_{(p,q)})} : \sqrt{(A : x_{(p,q)})} \text{ is an IF prime ideal of } M, x_{(p,q)} \in IFP(M)\}.$$

Theorem 4.7. *Let A be an IFI of a Noetherian Γ-ring M. Suppose $A = \bigcap_{i=1}^n Q_i$, be a minimal intuitionistic fuzzy primary decomposition of A. Let $P_i = \sqrt{Q_i}$, $1 \leq i \leq n$. Then $IF - ASS(A) = \{P_i, i = 1, 2, ..., n\}$ and these, are independent of the particular decomposition.*

Proof. Let $A = \bigcap_{i=1}^n Q_i$ with $P_i = \sqrt{Q_i}$, $1 \leq i \leq n$ be the minimal intuitionistic fuzzy primary decomposition of A. Consider any $x_{(p,q)} \in IFP(M)$, we have

$$(A : x_{(p,q)}) = (\bigcap_{i=1}^n Q_i : x_{(p,q)}) = \bigcap_{i=1}^n (Q_i : x_{(p,q)}). \text{ Hence } \sqrt{(A : x_{(p,q)})} = \bigcap_{i=1}^n \sqrt{(Q_i : x_{(p,q)})}.$$

Also, by Proposition (4.6), if $x_{(p,q)} \in Q_j$ then $\sqrt{(Q_j : x_{(p,q)})} = \chi_M$ and if, $x_{(p,q)} \notin Q_j$, then $\sqrt{(Q_j : x_{(p,q)})} = P_j$, be an intuitionistic fuzzy prime ideal of M. So

$$\sqrt{(A : x_{(p,q)})} = \bigcap_{i=1}^n \sqrt{(Q_i : x_{(p,q)})} = \bigcap_{x_{(p,q)} \notin Q_j} P_j.$$

Now, suppose that $P \in IF - ASS(A)$, then $P = \sqrt{(A : x_{(p,q)})}$ be an intuitionistic fuzzy prime ideal of M, for some $x_{(p,q)} \in IFP(M)$.
Since $\sqrt{(A : x_{(p,q)})} = \bigcap_{x_{(p,q)} \notin Q_j} P_j$, then by Lemma (4.1)(2) we have $\sqrt{(A : x_{(p,q)})} = P_j$ for some j. So, $P \in \{P_i, i = 1, 2, ..., n\}$. Therefore, $IF - ASS(A) \subseteq \{P_i, i = 1, 2, ..., n\}$.
Conversely, as the decomposition is minimal so $\bigcap_{j \neq i=1}^n Q_j \nsubseteq Q_i$. Then for each $i \in \{1, 2, ..., n\}$, there exists $(x_i)_{(p_i,q_i)} \in \bigcap_{j \neq i=1}^n Q_j$ and $(x_i)_{(p_i,q_i)} \notin Q_i$, we have

$$\sqrt{(A : (x_i)_{(p_i,q_i)})} = \bigcap_{j=1}^n \sqrt{(Q_j : (x_j)_{(p_j,q_j)})} = P_i$$

(Since all other's $\sqrt{(Q_j : (x_j)_{(p_j,q_j)})} = \chi_M$, for $j \neq i$ by Proposition (4.6)).
So, $P_i \in IF - ASS(A)$. Therefore, $\{P_i, i = 1, 2, ..., n\} \subseteq IF - ASS(A)$.
Hence, $IF - ASS(A) = \{P_i, i = 1, 2, ..., n\}$. Thus $IF - ASS(A)$ are independent of the particular decomposition. □

Example 4.3. Let $M = \Gamma = Z_{p_1^{n_1}} \times Z_{p_2^{n_2}} \times ..... \times Z_{p_k^{n_k}}$ be a commutative ring of order $n = p_1^{n_1}p_2^{n_2}...p_k^{n_k}$, where p_i are distinct primes. Then M is a Γ-ring. Let $M = \langle x_1, x_2, ..., x_k\rangle$ such that $o(x_i) = p_i^{n_i}$, for $1 \leq i \leq k$. Let $U_0 = \langle 0\rangle$, $U_1 = \langle x_1\rangle$, $U_2 = \langle x_1, x_2\rangle$,........,$U_k = \langle x_1, x_2, ..., x_k\rangle = M$ be the chain of ideals of M such that $U_0 \subset U_1 \subset .... \subset U_{k-1} \subset$

U_k.
Let A be any intuitionistic fuzzy ideal of M defined by

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in U_0 \\ \alpha_1 & \text{if } x \in U_1 \backslash U_0 \\ \alpha_2 & \text{if } x \in U_2 \backslash U_1 \\ \dots\dots\dots & \\ \alpha_k & \text{if } x \in U_k \backslash U_{k-1} \end{cases} ; \quad \nu_A(x) = \begin{cases} 0 & \text{if } x \in U_0 \\ \beta_1 & \text{if } x \in U_1 \backslash U_0 \\ \beta_2 & \text{if } x \in U_2 \backslash U_1 \\ \dots\dots\dots & \\ \beta_k & \text{if } x \in U_k \backslash U_{k-1}. \end{cases}$$

where $1 = \alpha_0 \geq \alpha_1 \geq ..... \geq \alpha_k$ and $0 = \beta_0 \leq \beta_1 \leq .... \leq \beta_k$ and the pair (α_i, β_i) are called double pins and the set $\wedge(A) = \{(\alpha_0, \beta_0), (\alpha_1, \beta_1), ..., (\alpha_k, \beta_k)\}$ is called the set of double pinned flags for the IFI A of M (by Theorem (2.5)).
Define IFSs A_i on M as follows:

$$\mu_{A_i}(x) = \begin{cases} 1, & \text{if } x \in M_i \\ \alpha_{i+1}, & \text{if otherwise} \end{cases} ; \quad \nu_{A_i}(x) = \begin{cases} 0, & \text{if } x \in M_i \\ \beta_{i+1}, & \text{otherwise.} \end{cases}$$

where $\alpha_i, \beta_i \in (0, 1)$ such that $\alpha_i + \beta_i \leq 1$, for $1 \leq i \leq k$ and $\alpha_{k+1} = \alpha_1$, $\beta_{k+1} = \beta_1$ and $M_i = Z_{p_1^{n_1}} \times ..... \times Z_{p_{i-1}^{n_{i-1}}} \times \langle 0 \rangle \times Z_{p_{i+1}^{n_{i+1}}} \times .... \times Z_{p_k^{n_k}}$ is a primary ideal of M. Clearly, A_i are intuitionistic fuzzy primary ideal of M. It can be easily checked that $A = \cap_{i=1}^n A_i$ is an intuitionistic fuzzy primary decomposition of A.

Example 4.4. Consider $M = \Gamma = \prod_{i=1}^{\infty} \mathbf{Z_2}$, a direct product of infinitely many copies of the field $\mathbf{Z_2} = \{\bar{0}, \bar{1}\}$ be a boolean ring. Then M is a Γ-ring, which is not a Noetherian ring, as the strictly ascending chain of ideals $\mathbf{0} \subset \mathbf{Z_2} \times \mathbf{0} \subset \mathbf{Z_2} \times \mathbf{Z_2} \times \mathbf{0} \subset ......$ is not stationary. For every $t_i, s_i \in [0, 1)$ such that $t_i + s_i \leq 1$, define $A_i \in IFS(M)$ as

$$\mu_{A_i}(x) = \begin{cases} 1, & \text{if } x = \prod_{i=1}^{\infty} \bar{0} \\ t_i, & \text{if otherwise} \end{cases} ; \quad \nu_{A_i}(x) = \begin{cases} 0, & \text{if } x = \prod_{i=1}^{\infty} \bar{0} \\ s_i, & \text{if otherwise.} \end{cases}$$

for all $x \in M$. Then by Theorem (2.2), A_i is an intuitionistic fuzzy prime ideal and hence primary ideal of M.

Consider the IFI A of M defined by $A(x) = (0, 1), \forall x \in M$. Then A has no intuitionistic fuzzy primary decomposition in M, i.e., $A \neq \bigcap_{i=1}^n A_i$, for any $n \in \mathbb{N}$.

Conclusion

In this paper, we introduced and studied the irreducibility of an intuitionistic fuzzy ideal of a Γ-ring. We proved that every intuitionistic fuzzy ideal A in a commutative Noetherian Γ ring can be decomposed as the intersection of a finite number of intuitionistic fuzzy primary ideals and established the intuitionistic fuzzy version of the Lasker-Noether theorem for a commutative Noetherian Γ ring. This decomposition is called an intuitionistic fuzzy primary decomposition. In addition to this, we have shown that, in case of minimal intuitionistic fuzzy primary decomposition of an intuitionistic fuzzy ideal A, the set of all intuitionistic fuzzy associated prime ideals of A, is independent of the particular decomposition. This primary decomposition theorem provides a generalization of the factorization of an ideal in a Noetherian ring as the intersection of a finite number of primary ideals. In a similar fashion, one can think of extending the intuitionistic fuzzy primary decomposition theorem to some other algebraic structures in spite of a commutative Γ-ring. From that point of view, our work on intuitionistic fuzzy primary decomposition in a commutative Noetherian Γ-ring sets up a new hierarchy, and surely it develops the study for some further research.

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REFERENCES

[1] Atanassov, K. T. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems.* **20** (1986), 87–96.
[2] Atanassov, K. T. *Intuitionistic Fuzzy Sets Theory and Applications.* Studies on Fuzziness and Soft Computing, 35, Physica-Verlag, Heidelberg, 1999.
[3] Barnes, W. E. On the Γ-rings of Nobusawa. *Pacific J. Math.* **18** (1966), 411–422.
[4] Coleman, R; Zwald, L. *Primary ideals*. *hal-03040606*, 2020.
[5] David, C. V. *Primary Decomposition in Boolean Rings*. arXiv:1707.07783v1 [math.AC] 25 Jul 2017.
[6] Kim, K. H.; Jun, Y. B; Öztürk, M. A. Intuitionistic fuzzy ideals of Γ-rings. *Sci. Math. Jpn.* **54** (2001), no. 1, 51-60.
[7] Kyuno, S. On prime gamma rings. *Pacific J. Math.* **75** (1978), no. 1, 185–190.
[8] Kyuno, S. Prime ideals in gamma rings. *Pacific J. Math.* **98** (1982), no. 2, 375–379.
[9] Nobusawa, N. On a generalization of the ring theory. *Osaka Math. J.* **1** (1964), 81–89.
[10] Paul, R. On various types of ideals of Γ-rings and the corresponding operator rings. *Int. J. of Eng. Res. and Appl.* **5** (2015), no. 8, 95–98.
[11] Palaniappan, N.; Veerappan, P. S.; Ramachandran, M. Characterization of Intuitionistic fuzzy ideals of Γ-rings. *Appl. Math. Sci.* **4** (2010), no. 23, 1107–1117.
[12] Palaniappan, N.; Veerappan, P. S.; Ramachandran, M. Some properties of Intuitionistic fuzzy ideals of Γ-rings. *Thai J. Math.* **9** (2011), no. 2, 305–318.
[13] Palaniappan, N.; Ramachandran, M. Intuitionistic fuzzy prime ideals in Γ-rings. *Int. J. of Fuzzy Math. and Sys.* **1** (2011), no. 2, 141–153.
[14] Ramachandran, M. *Some charcterization of intuitionistic fuzzy ideals in Γ-rings*. Ph. D., thesis submitted to Algappa University, 2010.
[15] Sharma, P. K.; Lata, H. Intuitionistic fuzzy characteristic ideals of a Γ-ring. *South East Asian J. Math. Math. Sci.* **18** (2022), no. 1, 49–70.
[16] Sharma, P. K.; Lata, H.; and Bhardwaj, N. Intuitionistic fuzzy prime radical and intuitionistic fuzzy primary ideal of Γ-ring. *Creat. Math. Inform.* **32** (2023), no. 1, 69–86.
[17] Sharma, P. K.; Lata, H.; and Bharadwaj, N. On Intuitionistic Fuzzy Structure Space On Γ-Ring. *Creat. Math. Inform.* **31** (2022), no. 2, 215–228.
[18] Warsi, Z. K. On decomposition of primary ideals of Γ-rings. *Department of Mathematics, University of Gorkhpur.* **9** (1978), no. 9, 912–917.

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