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Solving Fractional Gas Dynamic Equations with the Pythagorean Fuzzy Laplace Transform Iterative Method

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ABSTRACT. To solve homogeneous Pythagorean fuzzy fractional gas dynamic equations (PFFGDEs), this paper introduces the Pythagorean Fuzzy Laplace Transform Iterative Method (PF-LTIM), an effective technique. This work aims to close the gap between fractional dynamics and uncertainty. We can extract analytical answers in the form of simple-to-calculate series by utilising MATHEMATICA's computational capabilities. The remarkable agreement between these approximate solutions and the actual answers highlights the accuracy and effectiveness of the PF-LTIM.

1. INTRODUCTION

Gas dynamics is a field that studies the behavior of gases using mathematical models described by partial differential equations (PDEs)[4, 6, 5, 19, 28, 30, 35]. In recent years, there has been a growing interest in incorporating fuzzy logic and fractional calculus to handle systems with uncertainties and complex dynamics. The Pythagorean fuzzy fractional gas dynamic equation and Pythagorean fuzzy Laplace transform (PFLT) are two cutting-edge concepts that fusion together to provide insights into complex systems[1, 2]. The Pythagorean fuzzy Laplace transform is a transformative mathematical tool that combines fuzzy logic, fractional calculus, and the Laplace transform.

This study focuses on solving Pythagorean Fuzzy Fractional Gas Dynamic Equations (PFFGDEs) using the Pythagorean fuzzy Laplace transform iterative method, which combines fuzzy logic, fractional calculus, and the Laplace transform[1, 2, 11, 12, 13, 14, 20, 21, 42]. Further research can explore its application to more complex PFFGDEs and investigate its performance in other domains involving fuzzy and fractional characteristics.

Iterative algorithms are crucial for solving the Pythagorean fuzzy fractional gas dynamic equation, revealing complexities and extracting valuable insights[3, 8, 9, 15, 16, 17, 18, 26, 31, 38, 40, 43, 44]. The convergence of Pythagorean fuzzy fractional gas dynamics, Pythagorean fuzzy Laplace transform[1, 2], and iterative methods aims to unravel hidden relationships, discover patterns, and pave the way for groundbreaking applications across various disciplines.

The Pythagorean fuzzy Laplace transform iterative method demonstrates its capability to handle uncertainties and complex dynamics in gas systems[36, 37]. Researchers have proposed new similarity measures for Pythagorean fuzzy sets, explored dimensional analysis under linguistic Pythagorean fuzzy sets, and explored system reliability analysis based on Pythagorean fuzzy sets[11, 12, 13, 14, 20, 29, 32, 33, 39, 42]. Researchers tackle Pythagorean fuzzy fractional differential equations using Laplace transform and Atangana-Baleanu-Caputo fractional differential equations analytically using Pythagorean fuzzy sets[2], showcasing their applicability in solving fractional differential equations.

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Definition 1.1. [1, 39] *A Pythagorean fuzzy set (PFS)$\mathcal{P} \in \Delta$ in a non-empty setΔ is defined as follows*

$$\mathcal{P} = \{\langle x, W_{T_A}(x), W_{F_B}(x)\rangle / x \in \Delta\}, \tag{1.1}$$

where $W_{T_A}, W_{F_B} : \Delta \to [0,1]$ denotes object membership or non-membership degree $x \in \Delta$ respectively. The W_{T_A}, W_{F_B} must satisfy the following condition for all $x \in \Delta$:

$$0 \le \big(W_{T_A}(x)\big)^2 + \big(W_{F_B}(x)\big)^2 \le 1,$$

and the degree of hesitation represented by

$$\xi_* = \sqrt{1 - \big(W_{T_A}(x)\big)^2 - \big(W_{F_B}(x)\big)^2}$$

Definition 1.2. [1, 20, 21] *A Pythagorean fuzzy number (PFN)$P = \big[T_A, F_B\big]$ is a convex, concave, normal and non-empty subset of Δ with non-membership degree $W_{F_B} : \Delta \to [0,1]$ and membership degree $W_{T_A} : \Delta \to [0,1]$.*

Definition 1.3. [2] *A triangular Pythagorean fuzzy number(TPFN) P is a subset of PFN in $\mathbb{R}$ with membership and non-membership degrees*

$$W_{T_A}(x) = \begin{cases} \frac{x-p_1}{p_2-p_1}, & p_1 \le x \le p_2, \\ \frac{p_3-x}{p_3-p_2}, & p_2 \le x \le p_3, \\ 0, & otherwise \end{cases} \text{ and } W_{F_B}(x) = \begin{cases} \frac{p_2-x}{p_2-p_0}, & p_0 \le x \le p_2, \\ \frac{x-p_2}{p_4-p_1}, & p_2 \le x \le p_4, \\ 1, & otherwise \end{cases}$$

where $p_0 \le p_1 \le p_2 \le p_3 \le p_4$ and a TPFN is denoted by $P = \big[p_1, p_2, p_3; p_0, p_2, p_4\big]$.

Definition 1.4. [1, 27] *A quadruple*

$$\Big\langle \Big[P_{1(\alpha_\star;b)}, P_{2(\alpha_\star;b)}\Big]; \Big[P_{1(\beta_\star;t)}, P_{2(\beta_\star;t)}\Big]\Big\rangle$$

be a parametric Pythagorean fuzzy number (PPFN)P such that the functions $P_{1(\alpha_\star;b)}(x)$, $P_{2(\alpha_\star;b)}(x), P_{1(\beta_\star;t)}(x), P_{2(\beta_\star;t)}(x); x \in [0,1]$ and satisfy the following requirements:

- *$P_{1(\alpha_\star;b)}(x)$ and $P_{1(\beta_\star;t)}(x)$ is bounded, monotonically non-decreasing, right continuous at point 0 and left continuous on a set $(0,1]$.*
- *$P_{2(\alpha_\star;b)}(x)$ and $P_{2(\beta_\star;t)}(x)$ is bounded, monotonically non-increasing, right continuous at point 0 and left continuous on a set $(0,1]$.*
- *$P_{1(\alpha_\star;b)}(x) \le P_{2(\alpha_\star;b)}(x)$.*
- *$P_{1(\beta_\star;t)}(x) \le P_{2(\beta_\star;t)}(x)$.*

PPFNs with addition and multiplication operations are denoted by $\mathbb{F}$.

Definition 1.5. [1, 27] *The $(\alpha_\star, \beta_\star)-$ cut of membership and non-membership functions defined by*

$$\big[P\big]^{\big(\alpha_\star,\beta_\star;b,t\big)} = \Big\langle \Big[P_{1(\alpha_\star;b)}, P_{2(\alpha_\star;b)}\Big]; \Big[P_{1(\beta_\star;t)}, P_{2(\beta_\star;t)}\Big]\Big\rangle \; if \; P \in \mathbb{F} \tag{1.2}$$

where

$$\begin{aligned} P_{1(\alpha_\star;b)} &= \min\Big\{x/x \in \big[P\big]^{\big(\alpha_\star,\beta_\star;b,t\big)}\Big\} \\ P_{2(\alpha_\star;b)} &= \max\Big\{x/x \in \big[P\big]^{\big(\alpha_\star,\beta_\star;b,t\big)}\Big\} \\ P_{1(\beta_\star;t)} &= \min\Big\{x/x \in \big[P\big]^{\big(\alpha_\star,\beta_\star;b,t\big)}\Big\} \\ P_{2(\beta_\star;t)} &= \max\Big\{x/x \in \big[P\big]^{\big(\alpha_\star,\beta_\star;b,t\big)}\Big\} \end{aligned}$$

Definition 1.6. [1, 22] *Let $P^*, P^{**} \in \mathbb{F}$. The Hukahara difference between P^* and,P^{**} is defined by $P^* \ominus^H P^{**}$, if there exists P^{***} such that $P^* = P^{**} \oplus P^{***}$. Also $P^* \ominus^H P^{**} \neq P^* + \big(-P^{**}\big)$*

Definition 1.7. [1, 7] *Let P be a FVF and improper fuzzy Riemann integral(IFRI) $\int\limits_0^\infty e^{-st}P(t)dt$ is known as fuzzy Laplace transform(FLT) of FVF $P = \big[P_{1(\alpha_\star)}(t), P_{2(\alpha_\star)}(t)\big]$ and its symbolic notation is given by*

$$\mathcal{L}[P(t)] = \int\limits_0^\infty e^{-st}P(t)dt, p > 0 \tag{1.3}$$

Since

$$\int\limits_0^\infty e^{-st}P(t)dt = \Bigg[\int\limits_0^\infty e^{-st}P_{1(\alpha_\star)}(t)dt, \int\limits_0^\infty e^{-st}P_{2(\alpha_\star)}(t)dt\Bigg] \tag{1.4}$$

Using Equation (1.3) *we represent*

$$\mathcal{L}[P(t)] = \Big[l\big(P_{1(\alpha_\star)}(t)\big), l\big(P_{2(\alpha_\star)}(t)\big)\Big],$$

where

$$\begin{aligned} l\big(P_{1(\alpha_\star)}(t)\big) &= \int\limits_0^\infty e^{-st}P_{1(\alpha_\star)}(t)dt, \\ l\big(P_{2(\alpha_\star)}(t)\big) &= \int\limits_0^\infty e^{-st}P_{2(\alpha_\star)}(t)dt \end{aligned}$$

and $l\big(P(t)\big)$ represent the Laplace transform in the classical Mathematics. Note that $P(t)$ is a crisp function.

Definition 1.8. [2, 24] *Let $P \in \mathcal{C}^{\mathbb{F}} \cap \mathcal{L}^{\mathbb{F}}$ and improper Pythagorean fuzzy Riemann integral (IPFRI) $\int\limits_0^\infty e^{-st}P(t)dt$ is known as Pythagorean fuzzy Laplace transform(PFLT) of PFVF P and its symbolic notation is given by*

$$\mathcal{K}(p) = \mathcal{L}[P(t)] = \int\limits_0^\infty e^{-st}P(t)dt, p > 0 \tag{1.5}$$

Since

$$\int_0^\infty e^{-st}P(t)dt = \Big\langle \Big[\int_0^\infty e^{-st}P_{1(\alpha_\star)}(t)dt, \int_0^\infty e^{-st}P_{2(\alpha_\star)}(t)dt\Big];$$

$$\Big[\int_0^\infty e^{-st}P_{1(\beta_\star)}(t)dt, \int_0^\infty e^{-st}P_{2(\beta_\star)}(t)dt\Big]\Big\rangle \tag{1.6}$$

Using Equation (1.5) *we represent*

$$\mathcal{L}[P(t)] = \Big\langle \Big[l\big(P_{1(\alpha_\star)}(t)\big), l\big(P_{2(\alpha_\star)}(t)\big)\Big]; \Big[l\big(P_{1(\beta_\star)}(t)\big), l\big(P_{2(\beta_\star)}(t)\big)\Big]\Big\rangle,$$

where

$$l\big(P_{1(\alpha_\star)}(t)\big) = \int_0^\infty e^{-st}P_{1(\alpha_\star)}(t)dt,$$

$$l\big(P_{2(\alpha_\star)}(t)\big) = \int_0^\infty e^{-st}P_{2(\alpha_\star)}(t)dt$$

$$l\big(P_{1(\beta_\star)}(t)\big) = \int_0^\infty e^{-st}P_{1(\beta_\star)}(t)dt,$$

$$l\big(P_{2(\beta_\star)}(t)\big) = \int_0^\infty e^{-st}P_{2(\beta_\star)}(t)dt$$

and $l\big(P(t)\big)$ *represent the Laplace transform in the classical Mathematics. Note that* $P(t)$ *is a crisp function.*

Definition 1.9. [23, 25, 34, 41] *Let* $P \in \mathcal{C}^{\mathbb{F}} \cap \mathcal{L}^{\mathbb{F}}, \beta \in [0,1]$ *then the derivative in the sense of Pythagorean fuzzy Caputo- Fabrizio fractional order* β *concerning t as*

$$^{CF}\mathcal{D}_t^\beta P(t) = \Big\langle \Big[^{CF}\mathcal{D}_t^\beta P_{1(\alpha_\star)}(t), ^{CF}\mathcal{D}_t^\beta P_{2(\alpha_\star)}(t)\Big]; \Big[^{CF}\mathcal{D}_t^\beta P_{1(\beta_\star)}(t), ^{CF}\mathcal{D}_t^\beta P_{2(\beta_\star)}(t)\Big]\Big\rangle \tag{1.7}$$

where

$$^{CF}\mathcal{D}_t^\beta P_{1(\alpha_\star)}(t) = \frac{M(\beta)}{1-\beta}\int_\lambda^t P'_{1(\alpha_\star)}(W)e^{\frac{\beta(t-W)^\beta}{1-\beta}}dW,$$

$$^{CF}\mathcal{D}_t^\beta P_{1(\alpha_\star)}(t) = \frac{M(\beta)}{1-\beta}\int_\lambda^t P'_{2(\alpha_\star)}(W)e^{\frac{\beta(t-W)^\beta}{1-\beta}}dW,$$

$$^{CF}\mathcal{D}_t^\beta P_{1(\alpha_\star)}(t) = \frac{M(\beta)}{1-\beta}\int_\lambda^t P'_{1(\beta_\star)}(W)e^{\frac{\beta(t-W)^\beta}{1-\beta}}dW, \tag{1.8}$$

$$^{CF}\mathcal{D}_t^\beta P_{1(\alpha_\star)}(t) = \frac{M(\beta)}{1-\beta}\int_\lambda^t P'_{2(\beta_\star)}(W)e^{\frac{\beta(t-W)^\beta}{1-\beta}}dW$$

Definition 1.10. [23, 24, 25, 34] *Let $P \in \mathcal{C}^{\mathbb{F}} \cap \mathcal{L}^{\mathbb{F}}$, $\beta \in [0,1]$ and $M(\beta) > 0$, is the normalization function satisfying $M(1) = 1 = M(0)$ then the integral in the sense of Pythagorean fuzzy Caputo-Fabrizio fractional order β concerning t as*

$$ {}^{CF}\mathcal{I}_t^{\beta} P(t) = \left\langle \left[{}^{CF}\mathcal{I}_t^{\beta} P_{1(\alpha_\star)}(t), {}^{CF}\mathcal{I}_t^{\beta} P_{2(\alpha_\star)}(t)\right]; \left[{}^{CF}\mathcal{I}_t^{\beta} P_{1(\beta_\star)}(t), {}^{CF}\mathcal{I}_t^{\beta} P_{2(\beta_\star)}(t)\right] \right\rangle \tag{1.9}$$

where,

$$\begin{aligned}
{}^{CF}\mathcal{I}_t^{\beta} P_{1(\alpha_\star)}(t) &= \frac{1-\beta}{M(\beta)} P_{1(\alpha_\star)}(t) + \frac{\beta}{M(\beta)} \int_{\lambda}^{t} P_{1(\alpha_\star)}(\theta) d\theta, \\
{}^{CF}\mathcal{I}_t^{\beta} P_{2(\alpha_\star)}(t) &= \frac{1-\beta}{M(\beta)} P_{2(\alpha_\star)}(t) + \frac{\beta}{M(\beta)} \int_{\lambda}^{t} P_{2(\alpha_\star)}(\theta) d\theta; \\
{}^{CF}\mathcal{I}_t^{\beta} P_{1(\beta_\star)}(t) &= \frac{1-\beta}{M(\beta)} P_{1(\beta_\star)}(t) + \frac{\beta}{M(\beta)} \int_{\lambda}^{t} P_{1(\beta_\star)}(\theta) d\theta, \\
{}^{CF}\mathcal{I}_t^{\beta} P_{1(\alpha_\star)}(t) &= \frac{1-\beta}{M(\beta)} P_{2(\beta_\star)}(t) + \frac{\beta}{M(\beta)} \int_{\lambda}^{t} P_{2(\beta_\star)}(\theta) d\theta
\end{aligned} \tag{1.10}$$

Theorem 1.1. [1, 2] *Let $P \in \mathcal{C}^{\mathbb{F}} \cap \mathcal{L}^{\mathbb{F}}$. If $P(t)$ is PFCF differentiable, then the PFLT of PFCFD of P defined in defn of* (1.10) *of order $\beta \in (0,1]$ is defined as:*

$$\mathcal{L}\left[{}^{CF}\mathcal{D}_t^{\beta+n} P(t) \right] = \mathcal{L} \begin{bmatrix} {}^{CF}\mathcal{D}_t^{\beta+n} P_{1(\alpha_\star)}(t), \ {}^{CF}\mathcal{D}_t^{\beta+n} P_{2(\alpha_\star)}(t); \\ {}^{CF}\mathcal{D}_t^{\beta+n} P_{1(\beta_\star)}(t), \ {}^{CF}\mathcal{D}_t^{\beta+n} P_{2(\beta_\star)}(t) \end{bmatrix} \tag{1.11}$$

$$= \begin{bmatrix} l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{1(\alpha_\star)}(t) \right), \ l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{2(\alpha_\star)}(t) \right); \\ l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{1(\beta_\star)}(t) \right), \ l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{2(\beta_\star)}(t) \right) \end{bmatrix} \tag{1.12}$$

where

$$l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{1(\alpha_\star)}(t) \right) = \frac{1}{s+\beta(1-s)} \left\{ s^{n+1} l\left[P_{1(\alpha_\star)}(t) \right] - \sum_{k=0}^{n} s^{n-k} P_{1(\alpha_\star)}^{k}(0) \right\} \tag{1.13}$$

$$l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{2(\alpha_\star)}(t) \right) = \frac{1}{s+\beta(1-s)} \left\{ s^{n+1} l\left[P_{2(\alpha_\star)}(t) \right] - \sum_{k=0}^{n} s^{n-k} P_{2(\alpha_\star)}^{k}(0) \right\} \tag{1.14}$$

$$l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{1(\beta_\star)}(t) \right) = \frac{1}{s+\beta(1-s)} \left\{ s^{n+1} l\left[P_{1(\beta_\star)}(t) \right] - \sum_{k=0}^{n} s^{n-k} P_{1(\beta_\star)}^{k}(0) \right\} \tag{1.15}$$

$$l\left({}^{CF}\mathcal{D}_t^{\beta+n} P_{2(\beta_\star)}(t) \right) = \frac{1}{s+\beta(1-s)} \left\{ s^{n+1} l\left[P_{2(\beta_\star)}(t) \right] - \sum_{k=0}^{n} s^{n-k} P_{2(\beta_\star)}^{k}(0) \right\} \tag{1.16}$$

Definition 1.11. [10] *The Mittag-Leffler function is defined by*

$$E_\alpha(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)} \tag{1.17}$$

and

$$E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(\alpha k + \beta)} \tag{1.18}$$

2. Main result

This section presents a semi-analytic solution using the Pythagorean Fuzzy Laplace Transform and iterative transform method. The Laplace transform converts PFFGDEs into algebraic equations, utilizing Pythagorean fuzzy numbers to handle uncertainties. The iterative method refines initial guesses to obtain approximate solutions, providing a robust framework for solving PFFGDEs. Consider,

$$ {}^{CF}\mathcal{D}_t^{\beta}\nu(\xi,t) + \frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu(\xi,t)\big) + \tilde{\mathcal{F}}_2\big(\nu(\xi,t)\big) - P(x)\tilde{g}(\xi,t) = 0, \beta \in (0,1] \tag{2.19}$$

with initial condition

$$\nu(\xi,0) = k(x)G(\xi)$$

where

$${}^{CF}\mathcal{D}_t^{\beta}\nu(\xi,t) = \left\langle {}^{CF}\mathcal{D}_t^{\beta}\nu_{1(\alpha_\star)}(\xi,t), {}^{CF}\mathcal{D}_t^{\beta}\nu_{2(\alpha_\star)}(\xi,t); {}^{CF}\mathcal{D}_t^{\beta}\nu_{1(\beta_\star)}(\xi,t), {}^{CF}\mathcal{D}_t^{\beta}\nu_{2(\beta_\star)}(\xi,t) \right\rangle$$

is the Pythagorean fuzzy Caputo–Fabrizio derivative (PFCFD),

$$P(x) = \left\langle P_{1(\alpha_\star)}(x), P_{2(\alpha_\star)}(x); P_{1(\beta_\star)}(x), P_{2(\beta_\star)}(x) \right\rangle$$

is the the Pythagorean fuzzy valued function (PFVF),

$$\mathcal{F}_1\big(\nu(\xi,t)\big) = \left\langle \mathcal{F}_1\big(\nu_{1(\alpha_\star)}(\xi,t)\big), \mathcal{F}_1\big(\nu_{2(\alpha_\star)}(\xi,t)\big); \mathcal{F}_1\big(\nu_{1(\beta_\star)}(\xi,t)\big), \mathcal{F}_1\big(\nu_{2(\beta_\star)}(\xi,t)\big) \right\rangle$$

and

$$\mathcal{F}_2\big(\nu(\xi,t)\big) = \left\langle \mathcal{F}_2\big(\nu_{1(\alpha_\star)}(\xi,t)\big), \mathcal{F}_2\big(\nu_{2(\alpha_\star)}(\xi,t)\big); \mathcal{F}_2\big(\nu_{1(\beta_\star)}(\xi,t)\big), \mathcal{F}_2\big(\nu_{2(\beta_\star)}(\xi,t)\big) \right\rangle$$

represent the nonlinear terms $\nu_{\xi\xi}^2(\xi,t)$ and $\nu^2(\xi,t)$ of the unknown function of

$$\nu(\xi,t) = \left\langle \nu_{1(\alpha_\star)}(\xi,t), \nu_{2(\alpha_\star)}(\xi,t); \nu_{1(\beta_\star)}(\xi,t), \nu_{2(\beta_\star)}(\xi,t) \right\rangle,$$

$$k(x) = \left\langle k_{1(\alpha_\star)}(x), k_{2(\alpha_\star)}(x); k_{1(\beta_\star)}(x), k_{2(\beta_\star)}(x) \right\rangle$$

be the Pythagorean fuzzy number and $G(\xi)$ is the source term.
Employing Pythagorean Fuzzy Laplace Transform Eq. (2.19)

$$\mathcal{L}\left[{}^{CF}\mathcal{D}_t^{\beta}\nu(\xi,t)\right] + \mathcal{L}\left[\frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu(\xi,t)\big) + \tilde{\mathcal{F}}_2\big(\nu(\xi,t)\big) - P(x)\tilde{g}(\xi,t)\right] = 0, \tag{2.20}$$

(2.21)

$$\frac{1}{s+\beta(1-s)}\left[s\mathcal{L}\big[\nu(\xi,t)\big] - \nu(\xi,0)\right] + \mathcal{L}\left[\frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu(\xi,t)\big) + \tilde{\mathcal{F}}_2\big(\nu(\xi,t)\big) - P(x)\tilde{g}(\xi,t)\right] = 0,$$

$$\mathcal{L}\big[\nu(\xi,t)\big] = \frac{\nu(\xi,0)}{s} - \frac{\big(s+\beta(1-s)\big)}{s}\mathcal{L}\bigg[\frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu(\xi,t)\big) - \tilde{\mathcal{F}}_2\big(\nu(\xi,t)\big) + P(x)\tilde{g}(\xi,t)\bigg], \tag{2.22}$$

$$\mathcal{L}\big[\nu(\xi,t)\big] = \frac{k(x)G(\xi)}{s} - \frac{\big(s+\beta(1-s)\big)}{s}\mathcal{L}\bigg[\frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu(\xi,t)\big) - \tilde{\mathcal{F}}_2\big(\nu(\xi,t)\big) + P(x)\tilde{g}(\xi,t)\bigg], \tag{2.23}$$

taking inverse Laplace Transform of Equation (2.23) on both sides
(2.24)

$$\nu(\xi,t) = \mathcal{L}^{-1}\bigg[\frac{k(x)G(\xi)}{s}\bigg] - \mathcal{L}^{-1}\bigg[\frac{\big(s+\beta(1-s)\big)}{s}\mathcal{L}\bigg[\frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu(\xi,t)\big) - \tilde{\mathcal{F}}_2\big(\nu(\xi,t)\big) + P(x)\tilde{g}(\xi,t)\bigg]\bigg].$$

Assume that $\nu(\xi,t) = \sum\limits_{n=0}^{\infty}\nu_n(\xi,t)$ then Equation (2.24) can be written in recurrence form as

$$\nu_0(\xi,t) = \mathcal{L}^{-1}\bigg[\frac{k(x)G(\xi)}{s}\bigg] - \mathcal{L}^{-1}\bigg[\frac{\big(s+\beta(1-s)\big)}{s}\mathcal{L}\Big[P(x)\tilde{g}(\xi,t)\Big]\bigg] \tag{2.25}$$

$$\nu_{n+1}(\xi,t) = -\mathcal{L}^{-1}\bigg[\frac{\big(s+\beta(1-s)\big)}{s}\mathcal{L}\Big[\frac{1}{2}\tilde{\mathcal{F}}_1\big(\nu_n(\xi,t)\big) - \tilde{\mathcal{F}}_2\big(\nu_n(\xi,t)\big)\Big]\bigg]. \tag{2.26}$$

Hence, the exact or approximate solutions of the unknown function $\nu(\xi,t)$ are given by

$$\nu(\xi,t) = k(x)\sum_{n=0}^{\infty}\nu_n(\xi,t)$$

$$\nu(\xi,t) = \Big\langle k_{1(\alpha_\star)}(x), k_{2(\alpha_\star)}(x); k_{1(\beta_\star)}(x), k_{2(\beta_\star)}(x)\Big\rangle\sum_{n=0}^{\infty}\nu_n(\xi,t) \tag{2.27}$$

$$\nu_{1(\alpha_\star)}(\xi,t) = k_{1(\alpha_\star)}(x)\Big[\nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\dots\Big] \tag{2.28}$$

$$\nu_{2(\alpha_\star)}(\xi,t) = k_{2(\alpha_\star)}(x)\Big[\nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\dots\Big] \tag{2.29}$$

$$\nu_{1(\beta_\star)}(\xi,t) = k_{1(\beta_\star)}(x)\Big[\nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\dots\Big] \tag{2.30}$$

$$\nu_{2(\beta_\star)}(\xi,t) = k_{2(\beta_\star)}(x)\Big[\nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\dots\Big]. \tag{2.31}$$

2.1. **Convergence and Error Analyis.**

Theorem 2.2. *Let $\nu_p(\xi,t)$ and $\nu_n(\xi,t)$ be the members of the fuzzy Banach space H, and the exact solution of* (2.19) *be $\nu(\xi,t)$. The series solution $\sum\limits_{p=0}^{\infty}\nu_p(\xi,t)$ converges to $\nu(\xi,t)$, if $\nu_p(\xi,t) \le \lambda\nu_{p-1}(\xi,t)$ for $\lambda \in (0,1)$, that is for any $\nu > 0, \exists E$ such that $||\nu_{p+n}(\xi,t)|| \le \nu, \forall p,n > E$.*

Proof. Let $u_p(\xi,t) = \nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\ldots+\nu_p(\xi,t)$ be the sequence of p^{th} partial sums of the series $\sum\limits_{p=0}^{\infty}\nu_p(\xi,t)$. Now consider,

$$||u_{p+1}(\xi,t)-u_p(\xi,t)|| = ||\nu_{p+1}(\xi,t)|| \le \lambda||\nu_p(\xi,t)|| \le \lambda^2||\nu_{p-1}(\xi,t)|| \le \lambda^3||\nu_{p-2}(\xi,t)|| \vdots$$
$$\le \lambda^{p+1}||\nu_0(\xi,t)||, \forall n,p \in E.$$

Consider,

$$||u_p(\xi,t)-u_n(\xi,t)|| = ||\nu_{p+n}(\xi,t)|| = ||\big(u_p(\xi,t)-u_{p-1}(\xi,t)\big) + \big(u_{p-1}(\xi,t)-u_{p-2}(\xi,t)\big)+$$

$$\big(u_{p-2}(\xi,t)-u_{p-3}(\xi,t)\big)+\ldots+\big(u_{n+1}(\xi,t)-u_n(\xi,t)\big)||\leq||\big(u_p(\xi,t)-u_{p-1}(\xi,t)\big)||$$

$$+||\big(u_{p-1}(\xi,t)-u_{p-2}(\xi,t)\big)||+||\big(u_{p-2}(\xi,t)-u_{p-3}(\xi,t)\big)||+\ldots+||\big(u_{n+1}(\xi,t)-u_n(\xi,t)\big)||$$

$$\leq\lambda^p||\nu_0(\xi,t)||+\lambda^{p-1}||\nu_0(\xi,t)||+\lambda^{p-2}||\nu_0(\xi,t)||+\ldots+\lambda^{p-1}||\nu_0(\xi,t)||$$

$$=||\nu_0(\xi,t)||\big(\lambda^p+\lambda^{p-1}+\ldots+\lambda^{p+1}\big)=||\nu_0(\xi,t)||\big(\frac{1-\lambda^{p-n}}{1-\lambda}\big)\lambda^{n+1}.$$

Since $0<\lambda<1$, and $\nu_0(\xi,t)$ are bounded, so assume that,

$$\nu=||\nu_0(\xi,t)||\big(\frac{1-\lambda^{p-n}}{1-\lambda}\big)\lambda^{n+1},$$

we get the desired result. Also $\sum_{p=0}^{\infty}\nu_p(\xi,t)$ is a Cauchy sequence in H, which implies that there exists $\nu_0(\xi,t)\in H$ such that $\lim_{p\to\infty}\nu_p(\xi,t)=\nu(\xi,t)$, Hence prove. □

Theorem 2.3. *Let* $\sum_{p=0}^{q}\nu_p(\xi,t)$ *be the finite and approximate solution of* $\nu(\xi,t)$. *If* $||\nu_{p+1}(\xi,t)||\leq\lambda||\nu_0(\xi,t)||$ *for* $\lambda\in(0,1)$, *then the maximum absolute error is*

$$||\nu(\xi,t)-\sum_{p=0}^{q}\nu_p(\xi,t)||\leq\frac{\lambda^{q+1}}{1-\lambda}||\nu_0(\xi,t)||.$$

Proof.

$$||\nu(\xi,t)-\sum_{p=0}^{q}\nu_p(\xi,t)||=||\sum_{p=0}^{\infty}\nu_p(\xi,t)||\leq\sum_{p=q+1}^{\infty}||\nu_p(\xi,t)||$$

$$\leq\sum_{p=q+1}^{\infty}\lambda^q||\nu_0(\xi,t)||\lambda^{q+1}\big(1+\lambda+\lambda^2+\ldots\big)||\nu_0(\xi,t)||\leq\frac{\lambda^{q+1}}{1-\lambda}||\nu_0(\xi,t)||.$$

□

3. Application

Example 3.1. *Consider the Pythagorean fuzzy fractional differential equation*

$$^{CF}\mathcal{D}_t^{\beta}\nu(\xi,t)+\frac{1}{2}\big(\nu_\xi^2(\xi,t)\big)-\nu(\xi,t)\big[1-\nu(\xi,t)\big]=0,\beta\in(0,1] \tag{3.32}$$

with initial condition

$$\nu(\xi,0)=P(x)e^{-\xi}$$

where $P(x)=(\alpha_\star+4,6-\alpha_\star;5-2\beta_\star,5+2\beta_\star)$ *has the Pythagorean fuzzy exact solution*

$$\nu(\xi,t)=(\alpha_\star+4,6-\alpha_\star;5-2\beta_\star,5+2\beta_\star)e^{-\xi}E_\beta\big(t^\beta\big). \tag{3.33}$$

Using the the scheme of Equations (2.25) and (2.26), we obtain

$$
\begin{aligned}
\nu_0(\xi,t) &= P(x)e^{-\xi}\\
\nu_1(\xi,t) &= P(x)e^{-\xi}[-1+\beta-t\beta]\\
\nu_2(\xi,t) &= \frac{1}{2}P(x)e^{-\xi}\left[2+4(-1+t)\beta+(2-4t+t^2)\beta^2\right]\\
\nu_3(\xi,t) &= \frac{1}{6}P(x)e^{-\xi}\left[-6-18(-1+t)\beta-9(2-4t+t^2)\beta^2-(-6+18t-9t^2+t^3)\beta^3\right]\\
\nu_4(\xi,t) &= \frac{1}{24}P(x)e^{-\xi}\begin{bmatrix}24+96(-1+t)\beta+72(2-4t+t^2)\beta^2+\\16(-6+18t-9t^2+t^3)\beta^3+(24-96t+72t^2-16t^3+t^4)\beta^4\end{bmatrix}\\
\nu_5(\xi,t) &= \frac{1}{120}P(x)e^{-\xi}\begin{bmatrix}120(-1+\beta)^5-600t(-1+\beta)^4\beta+600t^2(-1+\beta)^3\beta^2-\\200t^3(-1+\beta)^2\beta^3+25t^4(-1+\beta)\beta^4-t^5\beta^5\end{bmatrix}\\
&\vdots
\end{aligned}
\tag{3.34}
$$

Similarly, we can obtain higher terms. The Series solution is obtained using Equation (2.27). Hence, we write the series solution

$$\nu(\xi,t) = P(x)\left[\nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\nu_3(\xi,t)+\ldots\right] \tag{3.35}$$

$$\nu(\xi,t) = (\alpha_\star+4, 6-\alpha_\star; 5-2\beta_\star, 5+2\beta_\star)\Big[\nu_0(\xi,t)+\nu_1(\xi,t)+\nu_2(\xi,t)+\nu_3(\xi,t)+\ldots\Big] \tag{3.36}$$

$$
\begin{aligned}
\nu_{1(\alpha_\star)}(\xi,t) &= -\frac{1}{120}(4+\alpha_\star)e^{-\xi}\beta\begin{bmatrix}t^5\beta^4-5t^4\beta^3(-4+5\beta)+\\20t^3\beta^2(7-16\beta+10\beta^2)-\\60t^2\beta(-6+21\beta-24\beta^2+10\beta^3)-\\120(3-6\beta+7\beta^2-4\beta^3+\beta^4)+\\120t(3-12\beta+21\beta^2-16\beta^3+5\beta^4)+\ldots\end{bmatrix}\\
\nu_{2(\alpha_\star)}(\xi,t) &= \frac{1}{120}(\alpha_\star-6)e^{-\xi}\beta\begin{bmatrix}t^5\beta^4-5t^4\beta^3(-4+5\beta)+\\20t^3\beta^2(7-16\beta+10\beta^2)-\\60t^2\beta(-6+21\beta-24\beta^2+10\beta^3)-\\120(3-6\beta+7\beta^2-4\beta^3+\beta^4)+\\120t(3-12\beta+21\beta^2-16\beta^3+5\beta^4)+\ldots\end{bmatrix}\\
\nu_{1(\beta_\star)}(\xi,t) &= \frac{1}{120}(5+2\beta_\star)e^{-\xi}\beta\begin{bmatrix}t^5\beta^4-5t^4\beta^3(-4+5\beta)+\\20t^3\beta^2(7-16\beta+10\beta^2)-\\60t^2\beta(-6+21\beta-24\beta^2+10\beta^3)-\\120(3-6\beta+7\beta^2-4\beta^3+\beta^4)+\\120t(3-12\beta+21\beta^2-16\beta^3+5\beta^4)+\ldots\end{bmatrix}\\
\nu_{2(\beta_\star)}(\xi,t) &= -\frac{1}{120}(5+2\beta_\star)e^{-\xi}\beta\begin{bmatrix}t^5\beta^4-5t^4\beta^3(-4+5\beta)+\\20t^3\beta^2(7-16\beta+10\beta^2)-\\60t^2\beta(-6+21\beta-24\beta^2+10\beta^3)-\\120(3-6\beta+7\beta^2-4\beta^3+\beta^4)+\\120t(3-12\beta+21\beta^2-16\beta^3+5\beta^4)+\ldots\end{bmatrix}
\end{aligned}
\tag{3.37}
$$

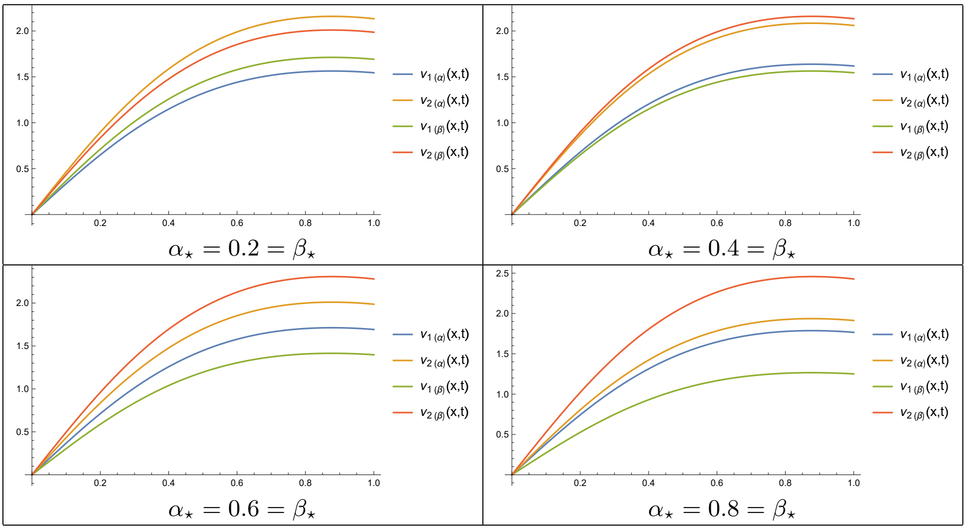

FIGURE 2. $(\alpha_\star, \beta_\star)-$cut simulation illustrating membership and non-membership functions for Equation (3.36).

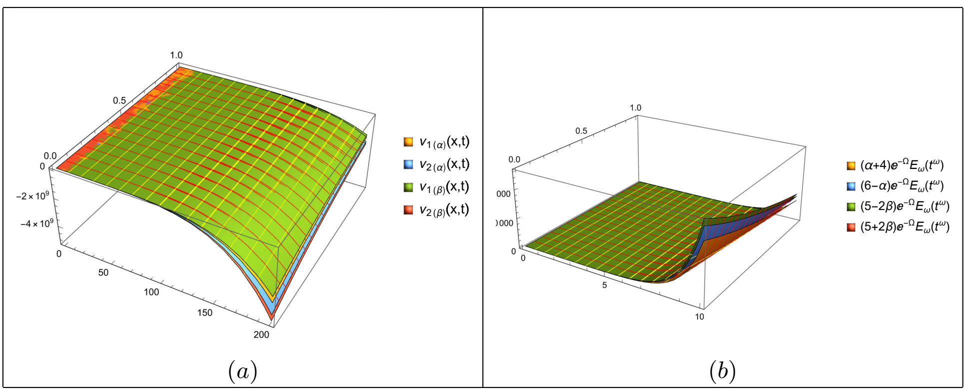

FIGURE 1. Three-dimensional visualization of (a) the fifth-order approximate numerical solution and (b) the exact solution of Equation (3.36).

Numerical experiments show the Pythagorean fuzzy Laplace transform iterative method effectively captures PFFGDEs, with approximate solutions agreeing well with exact solutions, validating the PF-LTIM table 1 accuracy.

4. CONCLUSIONS

This work shows that the Pythagorean Fuzzy Laplace Transform Iteration Method (PF-LTIM) may be used successfully to approximate solutions to Pythagorean fuzzy fractional gas dynamic equations. The PF-LTIM provides numerous significant benefits:

- The method effectively solves PFFGDEs without requiring time-consuming calculations utilizing Adomian polynomials, linearization, or perturbation assumptions.
- The PF-LTIM is simple to use, making it an ideal tool for researchers and practitioners.

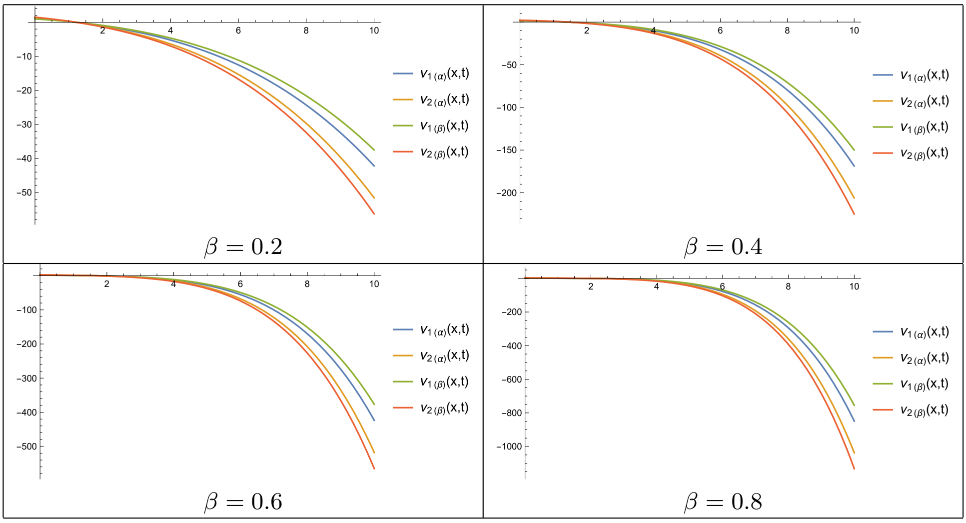

FIGURE 3. Two-dimensional representation of the solution of Equation (3.36) for various values of β

- The method gives accurate and efficient solutions for PFFGDEs, making it an important addition to the existing solution strategies.

The successful application of the PF-LTIM to PFFGDEs opens up new opportunities for future study in this field.

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TABLE 1. Error analysis between the approximate numerical solution and the exact solution of Equation (3.36).

t	Pythagorean Exact Solution $\nu(\xi,t)$				Pythagorean Approximate Solution $\nu(\xi,t)$				Error Analysis $\lvert Exact_{\nu(\xi,t)} - Approx_{\nu(\xi,t)}\rvert$			
t	$\nu_{1(\alpha_\star)}$	$\nu_{2(\alpha_\star)}$	$\nu_{1(\beta_\star)}$	$\nu_{2(\beta_\star)}$	$\nu_{1(\alpha_\star)}$	$\nu_{2(\alpha_\star)}$	$\nu_{1(\beta_\star)}$	$\nu_{2(\beta_\star)}$	$E_{1(\alpha_\star)}$	$E_{2(\alpha_\star)}$	$E_{1(\beta_\star)}$	$E_{2(\beta_\star)}$
0	2.72939	3.33592	2.42612	3.63918	2.72939	3.33592	2.42612	3.63918	0	0	0	0
0.1	3.01644	3.68676	2.68128	4.02192	2.46965	3.01846	2.19525	3.29287	0.546788	0.668296	0.486034	0.72905
0.2	3.33368	4.0745	2.96327	4.44491	2.23463	2.73122	1.98634	2.97951	1.09905	1.34328	0.976932	1.4654
0.3	3.68429	4.50302	3.27492	4.91238	2.02198	2.47131	1.79731	2.69597	1.66231	2.03171	1.47761	2.21641
0.4	4.07177	4.97661	3.61935	5.42902	1.82955	2.23612	1.62627	2.4394	2.24222	2.74049	1.99308	2.98963
0.5	4.5	5.5	4	6	1.6554	2.02327	1.47147	2.2072	2.8446	3.47673	2.52853	3.7928
0.6	4.97327	6.07844	4.42068	6.63103	1.49776	1.83059	1.33134	1.99701	3.47551	4.24785	3.08934	4.63402
0.7	5.49631	6.71772	4.88561	7.32842	1.35497	1.65607	1.20442	1.80663	4.14134	5.06164	3.68119	5.52179
0.8	6.07436	7.42422	5.39944	8.09915	1.2255	1.49784	1.08934	1.634	4.84886	5.92639	4.3101	6.46515
0.9	6.71321	8.20504	5.9673	8.95095	1.1079	1.35411	0.984804	1.47721	5.60531	6.85093	4.98249	7.47374
1	7.41925	9.06797	6.59489	9.89233	1.00078	1.22317	0.889578	1.33437	6.41847	7.8448	5.70531	8.55796

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