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Automorphisms and Structural Properties of Cayley Graphs on Matrix Monoids

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ABSTRACT. Let $M_n(K)$ be the set of all $n \times n$ matrices over a finite field K, and let G and E denote the group of invertible elements and idempotents in $M_n(K)$ respectively. We study the structure of the Cayley graph $Cay(M_n(K), G)$ and established that it is the disjoint union of complete graphs with automorphism group isomorphic to the direct product of symmetric groups. Furthermore, we investigate the existence of a bidirected arc in $Cay(M_n(K), E)$.

1. INTRODUCTION

A deep understanding of the combinatorial and geometric properties of algebraic structures like groups, semigroups, and monoids can be gained by studying Cayley graphs related to these structures. All $n \times n$ matrices over a finite field K constitute the full linear monoid $M_n(K)$, which serves as a complex structure connecting graph theory, linear algebra, and semigroup theory.

A lot of research has been done on Cayley graphs of groups, and some intriguing findings have been found. Cayley graphs of rectangular groups, brandt semigroups and inverse semigroups are characterized in [2], [8], [5] respectively. In [6], we investigated the Cayley graph of full transformation semigroups on a finite set with respect to the set of idempotents. Also, in [7] we examined the notion of traversability in the context of Rees matrix semigroups with respect to Green's equivalence $\mathcal{L}-$classes. By defining a recursion relation, explicit formulas for the eigenvalues of the rank-based Cayley graph on the additive group of matrices over a finite field are obtained in [4]. The significant influence of the girth of the transposition graph in determining the automorphism group of the associated Cayley graph was discussed in [1].

The study of Cayley graphs has traditionally focused on groups, with classical results describing their structure, automorphism groups, and spectral properties. In contrast, Cayley graphs of semigroups or monoids, particularly matrix monoids $M_n(K)$, have received comparatively little attention. Existing literature mainly concentrates on Cayley graphs arising from groups and on the algebraic structure of semigroups, such as Green's relations and decomposition theory, while the corresponding graph-theoretic properties remain largely unexplored. In particular, the structure of induced subgraphs associated with Green's L-classes, the characterization of their automorphism groups, and the spectral properties arising from these symmetries have not been systematically investigated. In this work, we bridge this gap by studying Cayley graphs on $M_n(K)$, establishing that each L-class induces a complete subgraph, determining the automorphism group of the Cayley graph in terms of symmetric groups, and analyzing the spectral properties of the associated permutation matrices. Furthermore, we extend these results to Cayley graphs

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generated by idempotent elements, thereby providing a unified graph-theoretic framework for understanding the internal structure of matrix monoids.

The Cayley graph of the full linear monoid illustrates a group through its generators and relations. The Cayley graph of a group is always vertex-transitive and regular. However, this is not always the case for monoids, where the presence of non-invertible elements and idempotents creates complex and highly structured graph topologies. This study reveals that the Cayley graph of the full linear monoid over a finite field decomposes into complete subgraphs, with its automorphism group closely reflecting the semigroup's $L - class$ structure.

2. Preliminaries

A digraph D is a pair (V, A), where V is a non-empty set whose elements are called the vertices of D and A is the set of ordered pairs of distinct elements of V, whose elements are called the directed edges or arcs of D. If for a pair of vertices u, v both (u, v) and (v, u) are arcs of D, then (u, v) and (v, u) are symmetric pair of arcs. Every arc in a symmetric digraph occurs as a symmetric pair. The digraph with vertex set V and each symmetric pair (uv) for $u, v \in V$ is called a complete symmetric digraph, or K_n^*, where $n = |V|$. Let D and D^* be digraphs. A mapping $f : V(D) \to V(D^*)$ is called a digraph homomorphism if $(u, v) \in A(D)$ implies $(f(u), f(v)) \in A(D^*)$, and is called a digraph isomorphism if it is bijective and both $f : D \to D^*$ and $f^{-1} : D \to D^*$ are graph homomorphisms. A graph isomorphism from D to itself is said to be an automorphism. We denote the set of all automorphisms on D by $\mathrm{Aut}(D)$.

An ordered pair $(S, *)$ where S is a non-empty set and $*$ is an associative binary operation on S is called a semigroup. A semigroup S with an identity is a monoid. The Green's relation L on S as $a\mathcal{L}b$ $(a, b \in S)$ if and only if a and b produce the same main left ideal, or if and only if $S^1a = S^1b$, where S^1 is the semigroup that is formed from S by, if required, adjoining an identity. Similarly the Green's relation R on S as $a\mathcal{R}b$ if and only if $aS^1 = bS^1$.

Lemma 2.1. [3] *Let a, b be elements of a semigroup S. Then $a\mathcal{L}b$ if and only if $\exists\, x, y \in S^1$ such that $xa = b$, $yb = a$ and $a\mathcal{R}b$ if and only if $\exists\, u, v \in S^1 : au = b$, $bv = a$.*

The $\mathcal{L}$-class (R-class) containing an element a in a semigroup S is denoted by $\mathcal{L}_a$ (R_a) and is defined as the set of all elements of S which are $\mathcal{L}$-equivalent (R-equivalent) to a, where $a \in S$.

Definition 2.1. *Let S be a finite semigroup and let H be a non-empty subset of S. The graph with vertex set S and arc set $\{(x, y) : hx = y \text{ for some } h \in H,\ x \neq y\}$ is known as the Cayley graph Cay(S, H) of S with respect to H.*

The set of all $n \times n$ matrices having non-zero determinant with entries from $\mathbb{Q}$ (rational numbers), $\mathbb{R}$ (real numbers), $\mathbb{C}$ (complex numbers), or $\mathbb{Z}_p$ (with p a prime) is a non-Abelian group under matrix multiplication, $n > 1$. This group is called the Full Linear Group of $n \times n$ matrices over $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, or $\mathbb{Z}_p$ (where p is a prime). If the entries are from K, where K is any of the above, we denote this group by $M_n(K)$. The full linear monoid encompasses all $n \times n$ matrices over a field (or division ring) K .

3. Main result

Lemma 3.2. *Let $M_n(K)$ be the set of all $n \times n$ matrices over a finite field K with group elements G. Then there exists an arc between a and b in Cay$(M_n(K), G)$ if and only if aLb.*

Proof. Let $a, b \in M_n(K)$ and assume an arc between a and b in Cay$(M_n(K), G)$. Then, by Definition 2.1, there exist $g_1, g_2 \in G$ such that $g_1 a = b$ and $g_2 b = a$. Therefore, by Lemma 2.1, we conclude that aLb.

Conversely, assume aLb. Then, by Definition 2.1, there exist $u, v \in G$ such that $ua = b$ and $vb = a$. Therefore, there is an arc between a and b in Cay$(M_n(K), G)$. □

Proposition 3.1. *Let G be the group elements of $M_n(K)$, and let L be any L-class of $M_n(K)$. Then the induced subgraph with vertex set L in the Cayley graph Cay$(M_n(K), G)$ is a complete graph K_m^*, where $m = |L|$.*

Proof. Let $a, b \in L$ with $a \neq b$. Then aLb. Hence, by Lemma 2.1, there is an arc between a and b in Cay$(M_n(K), G)$. Thus, there is an arc between a and b in the induced subgraph with vertex set L of Cay$(M_n(K), G)$.

Since a and b were chosen arbitrarily, it follows that there is an arc between every pair of distinct vertices in the induced subgraph with vertex set L. Also, since $|L| = m$, we conclude that the graph is K_m^*. □

Proposition 3.2. *Let G be the group of elements of $M_n(K)$, and let L be any L-class of $M_n(K)$. Then,*

$$Aut\,(Cay(M_n(K), G)[L]) \cong S_m,$$

where $m = |L|$, and Cay$(M_n(K), G)[L]$ denotes the induced subgraph of Cay$(M_n(K), G)$ with vertex set L.

Proof. From Proposition 3.1, we know that the induced subgraph Cay$(M_n(K), G)[L]$ is the complete graph K_m^*. The automorphism group of a complete graph K_m^* on m vertices is the symmetric group S_m, which consists of all possible permutations of the m vertices.

Since every vertex in K_m^* is adjacent to every other vertex, any bijection on the vertex set L preserves adjacency. Thus, every permutation of L corresponds to an automorphism of the graph. Conversely, any graph automorphism must map each vertex to another vertex while preserving adjacency, meaning it must be a permutation of L. Therefore,

$$\text{Aut}\,(\text{Cay}(M_n(K), G)[L]) \cong S_m.$$

□

Proposition 3.3. *Let G be the group of elements of $M_n(K)$, and let L be an L-class of $M_n(K)$ with $|L| = m$. Then, for each automorphism $\phi \in S_m$, there exists an associated automorphism matrix P_ϕ, an $m \times m$ permutation matrix satisfying the conjugation relation:*

$$P_\phi A_G P_\phi^{-1} = A_G.$$

Furthermore, the spectrum of any automorphism matrix P_ϕ consists of the eigenvalues

$$\lambda_k = e^{2\pi i k/m}, \quad k = 0, 1, \ldots, m-1.$$

Proof. By Proposition 3.2,

$$\text{Aut}(Cay(M_n(K), G)[L]) \cong S_m.$$

Each $\phi \in S_m$ induces a permutation of the vertex set L, which corresponds to an $m \times m$ permutation matrix P_ϕ satisfying

$$P_\phi P_\phi^T = I_m, \quad P_\phi^{-1} = P_\phi^T.$$

The adjacency matrix A_G of $Cay(M_n(K), G)[L]$ is given by

$$(A_G)_{ij} = \begin{cases} 1, & i \neq j, \\ 0, & i = j. \end{cases}$$

Since ϕ permutes the vertices of L, conjugation by P_ϕ preserves adjacency:

$$P_\phi A_G P_\phi^{-1} = A_G.$$

Using $P_\phi^{-1} = P_\phi^T$, it follows that

$$P_\phi A_G P_\phi^T = A_G.$$

Since P_ϕ represents a permutation, its eigenvalues correspond to the roots of unity. The permutation matrix can be decomposed into disjoint cycles, each contributing a circulant submatrix. A cycle of length k corresponds to a $k \times k$ matrix of the form

$$C_k = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

The characteristic polynomial of C_k is given by:

$$\det(C_k - \lambda I) = \lambda^k - 1.$$

Thus, the eigenvalues of C_k are the k^{th} roots of unity:

$$\lambda_j = e^{2\pi i j/k}, \quad j = 0, 1, \dots, k-1.$$

Since the full matrix P_ϕ is a direct sum of such cyclic submatrices, its eigenvalues are precisely the m^{th} roots of unity:

$$\lambda_k = e^{2\pi i k/m}, \quad k = 0, 1, \dots, m-1.$$

□

Theorem 3.1. *Let G be the group of elements of $M_n(K)$, and let $\{L_i\}_{i=1}^k$ be the collection of all L-classes of $M_n(K)$, where each L_i has cardinality $|L_i| = m_i$ for $i = 1, 2, \dots, k$. Then, the Cayley graph Cay$(M_n(K), G)$ is the disjoint union of k induced subgraphs, each containing m_i vertices*

$$\text{Cay}(M_n(K), G) = \bigcup_{i=1}^{k} \text{Cay}(M_n(K), G)[L_i].$$

Proof. Let L_i be an L-class of $M_n(K)$. By Proposition 3.1, the induced subgraph with vertex set L_i in Cay$(M_n(K), G)$ is the complete graph K_m^*, where $m = |L_i|$. Since L-classes are equivalence classes, they form a partition of $M_n(K)$, so

$$M_n(K) = \bigcup_{i=1}^{k} L_i,$$

and for any distinct $L_i, L_j \in M_n(K)$, we have $L_i \cap L_j = \emptyset$. Thus, the induced subgraphs with vertex sets L_i and L_j in Cay$(M_n(K), G)$ are disjoint. Since $i = 1, 2, \dots, k$, there exist k-arc disjoint complete subgraphs in Cay$(M_n(K), G)$.

To show that Cay$(M_n(K), G)$ is the disjoint union of these k-arc disjoint subgraphs, it suffices to prove that the arc set of Cay$(M_n(K), G)$ coincides with the arc set of the union of all induced subgraphs with vertex set L_i.

For this, let $x, y \in M_n(K)$. Suppose there is an arc from x to y in Cay$(M_n(K), G)$. Then, by Definition 2.1, there exists $g \in G$ such that $gx = y$, which implies $x = g^{-1}y$. By Lemma 2.1, we conclude that xLy, so $x, y \in L_i$ for some i. By Proposition 3.1, the induced subgraph with vertex set L_i in Cay$(M_n(K), G)$ is complete. Hence, the existence of an arc from x to y in the subgraph with vertex set L_i guarantees its presence in the union of all such induced subgraphs.

Conversely, assume there is an arc from x to y in the disjoint union of induced subgraphs with vertex set L_i of Cay$(M_n(K), G)$. Since the union is disjoint and each induced subgraph is complete, this arc must belong to a subgraph with vertex set L_i for some i. The partition

$$M_n(K) = \bigcup_{i=1}^{k} L_i$$

ensures that this arc also exists in Cay$(M_n(K), G)$.

Thus, the Cayley graph Cay$(M_n(K), G)$ is the disjoint union of k subgraphs with vertex counts $m_1, m_2, \ldots, m_k$. □

Example 3.1. *Let $M_2(\mathbb{Z}_2)$ be the set of all 2×2 matrices over the field $\mathbb{Z}_2$. Then $M_2(\mathbb{Z}_2) = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9, g_{10}, g_{11}, g_{12}, g_{13}, g_{14}, g_{15}, g_{16}\}$, where*

$$g_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad g_3 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad g_4 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

$$g_5 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad g_6 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad g_7 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad g_8 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

$$g_9 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad g_{10} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad g_{11} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad g_{12} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix},$$

$$g_{13} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad g_{14} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad g_{15} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad g_{16} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Now the group of units $GL_2(\mathbb{Z}_2)$ of $M_2(\mathbb{Z}_2)$ is given as $G = \{g_1, g_2, g_3, g_4, g_5, g_6\}$. Then the $Cay(M_2(\mathbb{Z}_2), G)$ is given in Fig 3.1.

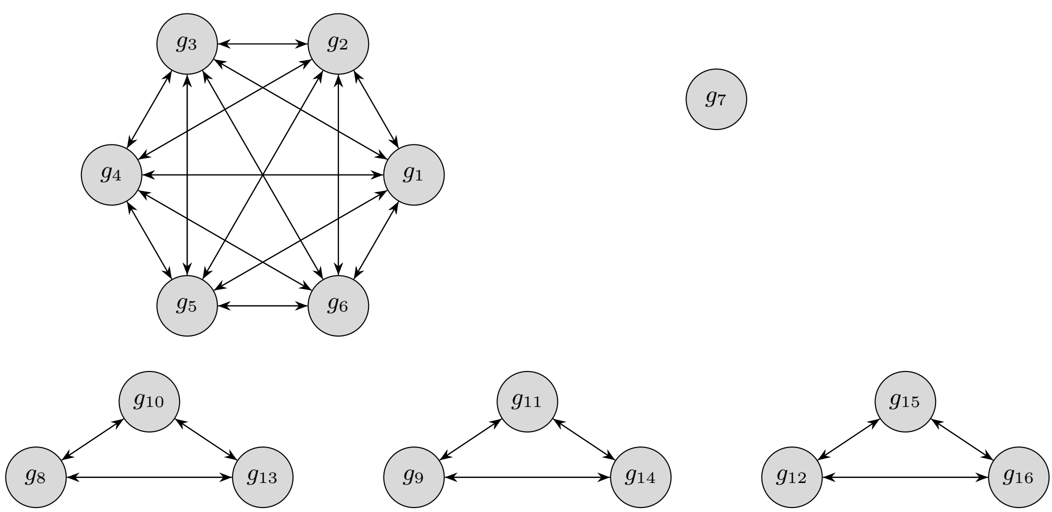

Fig: 3.1

Again the $\mathcal{L}$- classes of $M_2(\mathbb{Z}_2)$ other than G are $L_2 = \{g_7\}$, $L_3 = \{g_8, g_{10}, g_{13}\}$, $L_4 = \{g_9, g_{11}, g_{14}\}$ and $L_5 = \{g_{12}, g_{15}, g_{16}\}$. By Theorem 3.1, $Cay(M_2(\mathbb{Z}_2), G)$ is the union of these disjoint subgraphs, and each subgraph is a complete graph with vertices from the corresponding L-classes.

Theorem 3.2. *Let G be the group of elements of $M_n(K)$, and let $\{L_i\}_{i=1}^k$ be the distinct L-classes of $M_n(K)$, where each L_i has cardinality m_i. Then, the automorphism group of the Cayley graph Cay$(M_n(K), G)$ is the direct product of symmetric groups:*

$$Aut(Cay(M_n(K), G)) \cong \prod_{i=1}^{k} S_{m_i},$$

where all m_i are distinct.

Proof. Since each L_i forms a complete subgraph $K^*_{m_i}$ in Cay$(M_n(K), G)$, by Proposition 3.2, the automorphism group of the induced subgraph on L_i is precisely the symmetric group S_{m_i}, which permutes the m_i vertices freely. The set of L-classes $\{L_i\}_{i=1}^k$ partitions $M_n(K)$ into disjoint subsets, ensuring that each induced complete subgraph is independent of the others.

As a result, any automorphism of Cay$(M_n(K), G)$ must permute the vertices within each L_i independently while preserving adjacency relations. This independence implies that the full automorphism group of Cay$(M_n(K), G)$ is the direct product of the symmetric groups acting on each complete subgraph.

Thus, the automorphism group of Cay$(M_n(K), G)$ is given by

$$\text{Aut}(\text{Cay}(M_n(K), G)) \cong S_{m_1} \times S_{m_2} \times \cdots \times S_{m_k}.$$

□

Theorem 3.3. *Let G be the group of invertible elements in $M_n(K)$, and let $\{L_i\}_{i=1}^k$ be the L-classes of $M_n(K)$, where some classes have the same size. For each distinct size m_j, let r_j be the number of L-classes of that size. Then, the automorphism group of the Cayley graph $Cay(M_n(K), G)$ is:*

$$Aut(Cay(M_n(K), G)) \cong \left(\prod_j S_{m_j}^{r_j} \right) \rtimes S_{r_j}$$

where, $S_{m_j}^{r_j}$ acts on elements within each class of size m_j and S_{r_j} permutes these r_j classes.

Proof. By Theorem 3.2, when all m_i are distinct, the automorphism group of the Cayley graph $Cay(M_n(K), G)$ is given by

$$\text{Aut}(Cay(M_n(K), G)) \cong \prod_{i=1}^{k} S_{m_i},$$

since each L_i forms a complete subgraph $K^*_{m_i}$ and the automorphism group of the induced subgraph on L_i is precisely S_{m_i}, which permutes the m_i vertices freely. The distinctness of m_i ensures that each symmetric group acts independently, resulting in the direct product structure.

Now, if some L-classes have the same size, additional symmetries arise. Suppose there are r_j classes of size m_j. The automorphism group within each such class remains S_{m_j}, and since there are r_j independent classes of size m_j, the internal symmetries collectively form $S_{m_j}^{r_j}$, accounting for the independent permutations of elements within each class. Furthermore, since these r_j classes of size m_j are structurally identical, their labels can be permuted freely without affecting adjacency relations, introducing an additional symmetry group S_{r_j}, which permutes these r_j classes among themselves. Thus, the full automorphism group consists of the internal symmetries $S_{m_j}^{r_j}$ acting independently within each class, combined with the external symmetries S_{r_j}, which permute the r_j classes as a whole. This interaction results in a semi-direct product structure:

Aut$(Cay(M_n(K), G)) \cong \left(\prod_j S_{m_j}^{r_j}\right) \rtimes S_{r_j}$. □

Next, we discuss the Cayley graphs of $M_n(K)$ relative to set of idempotents.

Proposition 3.4. *Let $M_n(K)$ be the set of all $n \times n$ matrices over a finite field K, with group elements G and set of idempotents E. Then for $x, y \in M_n(K)$, there is a bidirected arc between x and y in the Cayley graph Cay$(M_n(K), E)$ if and only if xLy, provided $x, y \notin G$.*

Proof. Let $x, y \in M_n(K)$. Assume there is a bidirected arc between x and y in Cay$(M_n(K), E)$. Then there exist $e, e' \in E$ such that $ex = y$ and $e'y = x$. Then, by Lemma 2.1, xLy. Now suppose $x, y \in G$. Then $ex = y \Rightarrow exx^{-1} = yx^{-1} \Rightarrow e = yx^{-1} \Rightarrow e \in G$. Thus, e is the identity in G, which implies $x = y$, a contradiction. Conversely, suppose xLy with $x, y \notin G$. This implies that there exist $u, v \in M_n(K)$ such that $ux = y$ and $vy = x$. Suppose that there does not exist $e, e' \in E$ such that $ex = y$ and $e'y = x$. Thus, there exist $u, v \in M_n(K) \setminus E$ such that $ux = y$ and $vy = x$.

Let us consider R_u. Since every R-class contains an idempotent element, there exists some $f \in R_u$ such that $f \in E$. Also, since $f \in R_u$, we have fR_u. Therefore, by Lemma 2.1, there exist some $u_0, v_0 \in G$ such that $fu_0 = u$ and $uv_0 = f$. We know that $u_0x = x$ for some self-invertible, non-identity element $u_0 \in G$, and $x \in M_n(K) \setminus G$. Thus, $ux = fu_0x = fx$, which is a contradiction to our assumption. Hence, the proof is complete. □

Theorem 3.4. *Let $M_n(K)$ be the set of all $n \times n$ matrices over a finite field K with group elements G. Then the induced subgraph with vertex set L, other than G, of $Cay(M_n(K), E)$ is complete.*

Proof. Let L be an L-class of $M_n(K)$ other than G. Then $x, y \in L$ implies $x, y \notin G$. Hence, by Proposition 3.4, there exists a bidirected arc between x and y in the induced subgraph with vertex set L of $Cay(M_n(K), E)$. Since x and y are arbitrary, we have that the induced subgraph is K_m^*, where $m = |L|$. □

Example 3.2. *Let $M_2(\mathbb{Z}_2)$ be the set of all 2×2 matrices over the field $\mathbb{Z}_2$ and the set of idempotents $Idem(M_2(Z_2))$ is given as $E = \{g_7, g_8, g_9, g_{10}, g_{11}, g_{12}, g_{13}, g_{14}, g_{15}, g_{16}\}$. Then $Cay(M_2(Z_2), E)$ is given in Fig: 3.2*

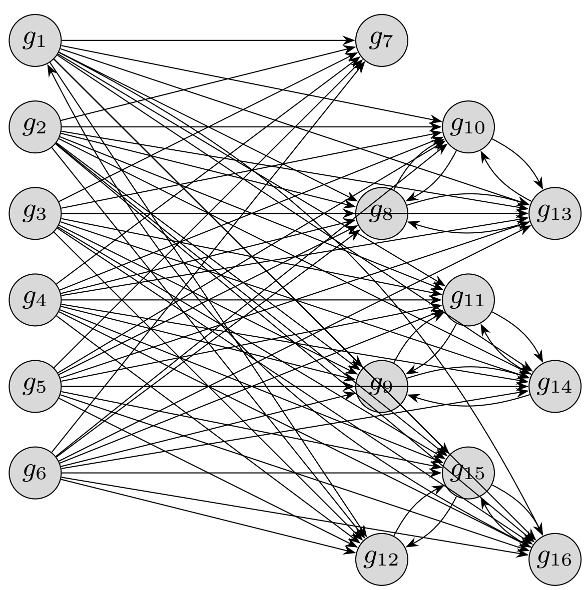

Fig: 3.2

By Theorem 3.4 the induced subgraph with vertex set L other than G of $Cay(M_2(Z_2), E)$ is complete.

4. Concluding Remarks

In this work, we investigated the structural properties of the Cayley graph associated with the full linear monoid $M_n(K)$ over a finite field K, with a focus on the automorphism group of the graph. We demonstrated that the Cayley graph $\mathrm{Cay}(M_n(K), G)$, where G is the group of invertible matrices, decomposes into a disjoint union of complete graphs determined by the $\mathcal{L}$-classes of $M_n(K)$. We further established that the automorphism group of this graph is isomorphic to a direct product or semidirect product of symmetric groups, depending on the multiplicity of class sizes. These findings contribute to a deeper understanding of the interplay between semigroup structure and graph symmetries, and lay the groundwork for further exploration of spectral, combinatorial, and algebraic properties of such graphs.

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