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Further Study on the s-Shunt Intersection Graph of a Graph

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ABSTRACT. For an integer $s \geq 1$, an *s-arc* in a graph G is a sequence of $(s+1)$ vertices $(v_1, v_2, ..., v_{s+1})$ of G such that any two consecutive vertices are adjacent in G and $v_i \neq v_{i+2}; 1 \leq i \leq s-1$. Certain structural properties of an intersection graph defined on the set of all s-arcs on distinct vertices of a graph G, that can be shunted onto another s-arc on distinct vertices of G, known as the s-shunt intersection graph of G is studied.

1. INTRODUCTION

We refer to [5] for basic terminology of graph theory and [7] for further concepts and results in intersection graph theory. Intersection graphs are graphs with a vertex set that is in a one-to-one correspondence with any family of sets and adjacency between any two vertices of the intersection graph exist if the intersection of the corresponding sets is non-empty. Every graph is an intersection graph of a family of subgraphs of a graph. For different types of intersection graphs, refer to [7].

For an integer $s \geq 1$, a sequence of $(s+1)$ vertices $(v_1, v_2, ..., v_{s+1})$ of a graph G such that any two consecutive vertices are adjacent in G and $v_i \neq v_{i+2}; 1 \leq i \leq s-1$, is an *s-arc* of G (see [4]). A vertex in an s-arc can be repeated even though this need not happen in all cases. Let $s \geq 1$ and $\alpha = (v_1, v_2, \ldots, v_{s+1})$ be an s-arc in a graph G. Henceforth, an s-arc $(v_1, v_2, \ldots, v_{s+1})$ will be represented as $v_1 v_2 \ldots v_{s+1}$. The head of the s-arc, denoted by $head(\alpha)$, is the $(s-1)$-arc $v_2 v_3 \ldots v_{s+1}$ and its tail, denoted by $tail(\alpha)$, is defined to be the $(s-1)$-arc $v_1 v_2 \ldots v_s$. It is important to note that as per the defintion of s-arc mentioned in [4], the s-arcs $v_1 v_2 \ldots v_{s+1}$ and $v_{s+1} v_s \ldots v_1$ are not considered equivalent since the head and tail of both of these s-arcs are distinct. An s-arc β follows α if there is an $(s+1)$-arc, say γ, such that $head(\gamma) = \beta$ and $tail(\gamma) = \alpha$. This can be described as α can be *shunted onto* β, which means α can be pushed one step onto β (see [4]). Note that shunting is the process by which trains are assembled, dissembled, sorted or positioned for journey. Hence, in a graph, s-arcs can be considered as trains or coaches that are shunted forward or backward.

Over the years, many intersection graphs derived from graphs have been introduced and studied. The *line graph* of a graph G, being one of the most studied intersection graphs derived from a graph, is an intersection graph on the set of all edges of G. Initially introduced in [10], line graphs have been defined and studied independently by many authors using different terminologies such as interchange graph, derived graph, derivative, covering graph, edge-to-vertex dual and adjoint. Different characterizations of line graphs was studied in [9] and [1]. One generalisation of the line graph of a graph is the *path graph* of a graph G (see [2]), denoted by $P_k(G)$, which is a graph whose vertices are k-paths of G and two distinct vertices of $P_k(G)$ are adjacent if the corresponding k-paths in G form a P_{k+1} or a C_k in G. Each path can be considered as a $(k-1)$-arc of G. If the union of two k-paths form a $(k+1)$-path, then this implies that one of the $(k-1)$-arcs was shunted

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onto the second $(k-1)$-arc, implying that some of the adjacencies of $P_k(G)$ are obtained based on the shuntability of one $(k-1)$-arc of G to another $(k-1)$-arc of G. Another similar graph introduced in [4] is the arc graph of a graph G, denoted by $X^s(G)$, which is a digraph with vertex set as the set of all s-arcs of G and a directed edge exists from α to β if α can be shunted onto β.

The notion of intersection graphs is widely used in many areas such as network analysis, flow problems and so on. Motivated by the above mentioned studies, the s-shunt intersection graph of a graph G was defined in [8] as follows.

Definition 1.1. Let S be the set of all s-arcs on distinct vertices in G. Then, the *s-shunt intersection graph* of G (in short, ssi-graph), denoted by $A_s(G)$, is the graph obtained as follows:

(i) An element $\alpha \in S$ belongs to $V(A_s(G))$ if there exists another s-arc $\beta \in S$ such that α can be shunted onto β on an $(s+1)$-arc on distinct vertices of G.
(ii) Two vertices α and β in $V(A_s(G))$ are adjacent in $A_s(G)$ if and only if their intersection in G is non-empty.

By a *shuntable s-arc* α of G, we mean that α is an s-arc of G that can be shunted onto some other s-arc of G. Henceforth, for all possible values of s, any s-arc mentioned is an s-arc with no repeated vertices and $s \geq 1$.

Example 1.1. Observe that the 3si-graph of the graph in Figure 2 has 45 vertices. For the 3-arc $u_1u_2u_3u_5$ of the graph in Figure 2, the head and tail are $head(u_1u_2u_3u_5) = u_2u_3u_5$ and $tail(u_1u_2u_3u_5) = u_1u_2u_3$. Also, there exists a 4-arc $u_1u_2u_3u_5u_6$ on distinct vertices in G with $tail(u_1u_2u_3u_5u_6) = u_1u_2u_3u_5$ and $head(u_1u_2u_3u_5u_6) = u_2u_3u_5u_6$. Therefore, the 3-arc $u_1u_2u_3u_5$ is shuntable and can be shunted onto the 3-arc $u_2u_3u_5u_6$. As per Definition 1.1, it follows that $u_1u_2u_3u_5$ is a vertex of $A_3(G)$. In contrast, the 3-arcs $u_4u_5u_1u_6$ and $u_6u_1u_7u_8$ are not vertices of $A_3(G)$ since they cannot be shunted onto any other 3-arc of G along a 4-arc with distinct vertices of G. Additionally, note that the 7-shunt intersection graph does not exist for the graph in Figure 2 since there does not exist any 7-arc α in G that can be shunted onto some other 7-arc β in G, such that $\alpha \cup \beta$ is an 8-arc of G.

The upper bound of s as per Definition 1.1 depends on the graph classes under consideration. The *detour distance* of a graph G, denoted by $D(u,v)$, is the length of the longest path between two vertices u and v of G (ref. [5]). The *detour eccentricity* $e_D(v)$ of a vertex v is the detour distance from v to a vertex farthest from v. The maximum detour eccentricity among all vertices of G is the *detour diameter* of a graph which is denoted as $diam_D(G)$ (ref. [3]). For ease of usage, we let s^* be the detour diameter of a graph G. Note that for any connected graph G, the ssi-graph is defined for integers $1 \leq s \leq s^* - 1$. Also, if G is a tree, then $s^* = diam(G)$.

For any graph G and any possible value of s, if $\alpha_i = u_iu_{i+1}\ldots u_{i+s}$ is an s-arc of G, then the sequence obtained by reversing the order of the vertices of α_i, denoted by $\alpha_i' = u_{i+s}u_{i+s-1}\ldots u_i$ is also an s-arc of G, that is distinct from α_i. By the left-end vertex and the right end-vertex of α_i, we mean the vertices u_i and u_{i+s} of α_i respectively. Unless mentioned otherwise, the graphs are simple, connected, undirected and finite. It is important to note that for possible values of s, the ssi-graph of a connected graph G is always connected (ref. [8]).

A preliminary structural analysis of the ssi-graph of a graph has been carried out in the literature (see [8]), where certain basic properties and characterizations were established. Specific graphs whose ssi-graphs are chordal were identified in [8], providing new insights into the chordality of the ssi-graph of a graph. Building on these observations, the present paper develops further structural properties of the ssi-graph of a graph G, with particular emphasis on conditions that facilitate a deeper and more systematic study of its chordality. The following results established in [8] identify a few chordal graphs that are the ssi-graph of some graph G.

Proposition 1.1. [8] *For any connected graph G of order n, $A_s(G)$ is complete when* $s \geq \left\lfloor \frac{n}{2} \right\rfloor$.

Lemma 1.1. [8] *The $(s^* - 1)$-shunt intersection graph of G is a complete graph.*

Theorem 1.1. [8] *The only acyclic graph that is realisable as the ssi-graph of a graph is K_2. Moreover, the ssi-graph of a graph G is acyclic if and only if there exists a unique s^*-arc in G.*

Theorem 1.2. [8] *The only cycle realisable as the ssi-graph of a graph is C_3.*

We first determine those graphs whose ssi-graphs is isomorphic to C_3.

Proposition 1.2. *Every vertex in G determines a clique in $A_s(G)$.*

Proof. Since the intersection of vertices of $A_s(G)$ that contains a vertex u of G is non-empty, it follows that a clique is induced in $A_s(G)$. □

Proposition 1.3. *The ssi-graph of a graph G is a C_3 if and only if G contain exactly two $(s+1)$-arcs α and β such that $tail(\alpha) = tail(\beta)$.*

Proof. Suppose α_1 and α_2 are the first two vertices of $A_s(G)$ which is a 3-cycle. Note that since α_1 and α_2 are vertices of $A_s(G)$, both α_1 and α_2 can be shunted onto some other s-arc of G, say β_1 and β_2 respectively. This implies that β_1' and β_2' are vertices of $A_s(G)$ irrespective of whether β_1 and β_2 are vertices of $A_s(G)$. But since $A_s(G)$ has exactly three vertices, G contains exactly two s-arcs that can be shunted onto the same s-arc of G (or G contains one s-arc that can be shunted onto two different s-arcs of G). Hence, if $A_s(G)$ is C_3, it is necessary that G contains exactly two $(s+1)$-arcs with a common tail, where the detour diameter of the graph is $s+1$. It follows that if G satisfies the given conditions, then certainly $A_s(G) \cong C_3$. □

Corollary 1.1. *The 1si-graph of a graph G is realizable as C_3 if and only if G is $K_{1,3}$.*

Proof. An immediate consequence of Proposition 1.3 is that $K_{1,3}$ contains exactly two 1-arcs with a common tail. Hence, $A_1(K_{1,3})$ is a C_3. Moreover, if G is not a $K_{1,3}$, either $A_1(G)$ is a K_2 or a graph with more than 3 vertices. □

Corollary 1.2. *The 2si-graph of a graph G is realizable as a C_3 if and only if G is a subdivision of $K_{1,3}$ where exactly one edge of $K_{1,3}$ is subdivided only once.*

Proof. The result is immediate from Proposition 1.3 and the arguments mentioned in the proof of Corollary 1.1. □

For higher values of s, there are graphs other than trees such that their ssi-graph is C_3. Obtaining a family of graphs such that their ssi-graph is C_3 is currently under study. As seen earlier, the only acyclic graph that is realisable as the ssi-graph of a graph G is K_2 (see Theorem 1.1). Also, if $A_s(G)$ is a graph on three vertices, then $A_s(G)$ is C_3. But for graphs where the order of $A_s(G)$ is greater than 3, can $A_s(G)$ have pendant vertices? The following result proves that the only graphs with pendant vertices that is realisable as the ssi-graph of any graph G is K_2.

Theorem 1.3. *If $|V(A_s(G))| \geq 3$, then $A_s(G)$ does not have pendant vertices.*

Proof. Let α be a pendant vertex of $A_s(G)$. Since $\alpha = u_iu_{i+1}\dots u_{i+s}$ is a vertex of $A_s(G)$, α can be shunted onto some other s-arc of G, say $\beta = u_{i+1}u_{i+2}\dots u_{i+s+1}$. A consequence of Definition 1.1 is that $\alpha \sim \beta'$. But since α is a pendant vertex of $A_s(G)$, the only neighbour of α is β' implying that there exists no other $(s+1)$-arc in G, other than β, such that α is the tail of the $(s+1)$-arc $u_iu_{i+1}\dots u_{i+s}u_{i+s+1}$ and β is not a vertex of $A_s(G)$. Since the order of $A_s(G)$ is greater than 2 and $A_s(G)$ is connected, β' is adjacent to some other vertex of $A_s(G)$, say γ, which implies that u_{i+s+1} is the vertex of β' that is not a vertex of α. Since $\gamma \nsim \alpha$ and $\gamma \sim \beta$, u_{i+s+1} is the only vertex in G that is in $V(\beta) \cap V(\gamma)$. This implies that β can be shunted onto another s-arc of G implying that α is adjacent to both β and β', a contradiction. □

Recall that a graph G is said to be *chordal* if either G is acyclic or G does not contain induced cycles of length greater than 3. The following result estbalishes a few conditions for the ssi-graph of a graph G to be chordal.

Lemma 1.2. *The ssi-graph of a graph G is chordal if any of the following conditions is true:*

(i) $s = s^* - 1$.
(ii) $s \geq \lfloor \frac{n}{2} \rfloor$.
(iii) *G is a tree;* $s \geq 1$.
(iv) *circumference of G is* 3; $s \geq 1$.

Proof. In view of Proposition 1.1 and Lemma 1.1, if $s = s^* - 1$ or $s \geq \lfloor \frac{n}{2} \rfloor$, then $A_s(G)$ is complete and hence chordal. Let G be a graph with circumference k for which $A_s(G)$ contains an induced C_p where $p > k$. Let $\alpha_i = v_iv_{i+1}\dots v_{i+s}; 1 \leq i \leq p$ be the vertices of an induced C_p in $A_s(G)$. Since $\alpha_2 \sim \alpha_1$, $\alpha_2 \sim \alpha_3$ and α_2 is not adjacent to any other vertices in C_p of $A_s(G)$, the vertices in α_1 and α_3 are distinct. There exists at least one vertex in the s-arc α_2, say a_1 in G, that is a vertex in the s-arc α_1, and another vertex in the s-arc α_2, say a_2, that is also a vertex of α_3. Since G is connected, there exists a path in G with a_1 and a_2 as the end-vertices. Similarly, there exists at least one vertex a_3 that is common to the s-arcs α_3 and α_4 and since G is connected, there is a path in G with a_2 and a_3 as the end-vertices. Continuing this way, there is a path in G with a_1 and a_{p-1} as the end-vertices. Since $\alpha_p \sim \alpha_1$, there exists a vertex a_p in G that belongs to the intersection of $V(\alpha_p)$ and $V(\alpha_1)$. It follows that G contains a cycle of order at least $p > k$ with vertices $a_1 \dots a_2 \dots a_3 \dots a_p \dots a_1$ which is a contradiction. Since the order of the largest induced cycle in a chordal graph is 3, it follows that if the circumference of G is 3, then the ssi-graph of such graphs is always chordal. Using the same argument for a tree, it follows that the ssi-graph of a tree $T_2(4,1)$ cannot have an induced cycle of order greater than 3. □

In view of the arguments mentioned in the proof of Lemma 1.2, the following results can be inferred.

Corollary 1.3. *The ssi-graph of a graph G with circumference k is* C_p*-free for every* $p \geq k+1$.

Corollary 1.4. *The largest induced hole in the ssi-graph of a tree is* C_3.

The converse of Lemma 1.2 is not generally true because there are chordal graphs that are ssi-graphs of some graph G, where the circumference of G is greater than 3. For example, the 3si-graph of the graph $C_5 \odot K_1$ is chordal even though the circumference of $C_5 \odot K_1$ is greater than 3, $s^* = 6$ and $s = 3 < \lfloor \frac{n}{2} \rfloor$. The following result describes one of the conditions for the existence of an induced k-cycle in $A_s(G)$.

Theorem 1.4. *A cycle of length k is an induced subgraph of* $A_s(G)$ *if G contains a p-cycle such that*

(i) $\frac{k}{2}(s+1) \leq p \leq ks$, *when k is even;*

(ii) $\frac{k+1}{2}s + \frac{k-1}{2} \leq p \leq ks$, *when* k *is odd.*

Proof. Let $\alpha_i = v_i v_{i+1} \dots v_{i+s}$ be an s-arc of a cycle C_k in G which is a vertex of $A_s(G)$. The vertex α_i is adjacent to the vertices $\alpha_{i-s}, \dots, \alpha_{i-2}, \alpha_{i-1}, \alpha_{i+1}, \alpha_{i+2}, \dots, \alpha_{i+s}, \alpha'_{i-s}, \dots, \alpha'_{i-2}$, $\alpha'_{i-1}, \alpha'_i, \alpha'_{i+1}, \alpha'_{i+2}, \dots,$ α'_{i+s} subject to the existence of these vertices in $V(A_s(G))$. Observe that there may be other s-arcs in G that are vertices of $A_s(G)$ which are adjacent to α_i. Since α_{i+1} is a neighbour of α_i in $A_s(G)$, the other neighbour of α_{i+1} which is not adjacent to α_i in $A_s(G)$ is α_{i-s}. Similarly, α_{i+2} and α_{i-s} (or $\alpha_{i-(s-1)}$) are neighbours of α_i in $A_s(G)$ that are not adjacent to each other. If $A_s(G)$ contains an induced cycle of order k, following the construction pattern mentioned in Lemma 1.2, the maximum order of a cycle (not necessarily induced) in G is ks. The vertices of the induced cycle in this case are $\alpha_i\alpha_{i+s}\alpha_{i+2s}\ \alpha_{i+3s} \dots \alpha_{i+(k-1)s}$, where $\alpha_{i+(k-1)s} = v_{i+(k-1)s}v_{i+(k-1)s+1} \dots v_{i+ks-1}v_i$. The minimum order of a cycle in G such that a k-cycle is induced in $A_s(G)$ is k. However, the minimum order of a k-cycle induced in $A_s(G)$ where all the vertices of the induced k-cycle of $A_s(G)$ are s-arcs of the same cycle in G is $\frac{k}{2}(s+1)$if k is even and $\frac{k+1}{2}s + \frac{k-1}{2}$ if k is odd. The vertex sets of the induced C_k for both these cases are $\alpha_i\alpha_{i+1}\alpha_{i+s+1}\alpha_{i+s+2}\alpha_{i+2s+2}$ $\alpha_{i+2s+3} \dots \alpha_{i+(\frac{k}{2}-1)s+(\frac{k}{2}-1)}\alpha_{i+(\frac{k}{2}-1)s+\frac{k}{2}}$ and $\alpha_i\alpha_{i+1}\alpha_{i+s+1}\alpha_{i+s+2}\alpha_{i+2s+2}\alpha_{i+2s+3}\alpha_{i+3s+3}$ $\alpha_{i+3s+4} \dots \alpha_{i+(\frac{k-1}{2}-1)s+\frac{k-1}{2}}\alpha_{i+\frac{k-1}{2}s+\frac{k-1}{2}}$ respectively, where $\alpha_{i+(\frac{k}{2}-1)s+\frac{k}{2}} = \{v_{i+(\frac{k}{2}-1)s+\frac{k}{2}},$ $v_{i+(\frac{k}{2}-1)s+\frac{k}{2}+1}, \dots, v_{i+\frac{k}{2}s+\frac{k}{2}-1}, v_i\} \subseteq V(G)$ and $\alpha_{i+\frac{k-1}{2}s+\frac{k-1}{2}} = \{v_{i+\frac{k-1}{2}s+\frac{k-1}{2}}, v_{i+\frac{k-1}{2}s+\frac{k-1}{2}+1},$ $\dots, v_{i+\frac{k-1}{2}s+\frac{k-1}{2}+s-1}, v_i\} \subseteq V(G)$. □

Corollary 1.5. *A cycle of length at least* 4 *is an induced subgraph of* $A_s(G)$ *if* G *contains a cycle of length* $2s+2$ *or higher.*

Proof. It follows from Theorem 1.4 that for every $k \geq 2s+2$, there exists an induced p-cycle in $A_s(G)$ where $p \geq 4$. □

Proposition 1.4. *If the ssi-graph of a graph G is chordal, then one of the following conditions is true:*

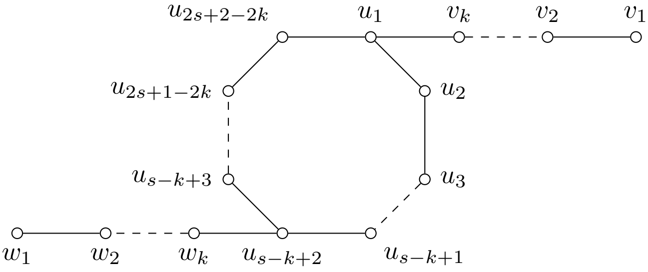
(i) $s = s^* - 1$
(ii) $s \geq \lfloor \frac{n}{2} \rfloor$
(iii) *G is a tree;* $s \geq 1$.
(iv) *circumference of G is at most* $2s+1$; $s \geq 1$.

Proof. Suppose that $A_s(G)$ is chordal. This implies that the $A_s(G)$ is a tree or the only induced cycle in $A_s(G)$ is a 3-cycle. If $A_s(G)$ of G is a complete graph, then $s = s^* - 1$ or $s \geq \lfloor \frac{n}{2} \rfloor$. It is clear from Corollary 1.5 that the circumference of G is at most $2s+1$. To prove (iii), assume that the ssi-graph of a graph G which contains a k-cycle, where $k > 2s+1$, is chordal. This implies that G is either a tree or the circumference is at least $2s+2$. As a consequence of Corollary 1.5, it follows that G is a tree. □

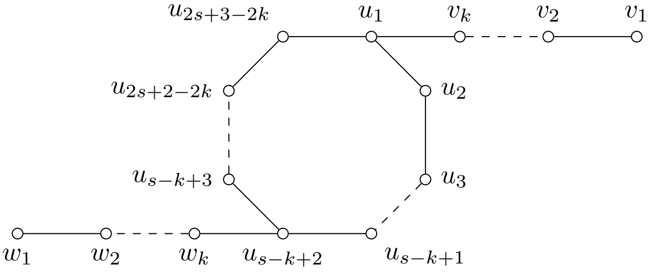
Interestingly, for $s \geq 2$, there exist graphs with k-cycles where $4 \leq k \leq 2s+1$, such that their ssi-graph is not chordal. It is enough to find the conditions for which a C_4 is induced in $A_s(G)$ since higher the order of a cycle in G, higher the order of an induced cycle in $A_s(G)$. The following result, even though not rigorous, provides certain conditions on G for which the corresponding ssi-graph is not chordal.

A *tadpole graph*, denoted by $T(m,n)$, is a graph obtained by attaching an n-path to an n-cycle by a bridge (ref. [6]). A *twin-tailed tadpole graph*, denoted by $T_2(m,n)$, is a graph obtained by attaching a copy of an n-path to each of two vertices of an m-cycle, which are at diametral distance with each other, by a bridge (see Figure 1 for illustration).

Lemma 1.3. *If the ssi-graph of a graph G is chordal, then G does not contain the following subgraphs:*



(A) $T_2(2s+2-2k,k)$



(B) $T_2(2s+3-2k,k)$

FIGURE 1. Examples for twin-tailed tadpole graphs.

(i) cycle C_k*;* $s \geq 1; k \geq 2s+2$*,*
(ii) tadpole graph $T(2s+2-k,k)$*;* $s \geq 2; 1 \leq k \leq s-1$*, and*
(iii) twin-tailed tadpole graph $T_2(2s+2-2k,k)$*;* $s \geq 2; 1 \leq k \leq s-1$*.*
(iv) twin-tailed tadpole graph $T_2(2s+3-2k,k)$*;* $s \geq 2; 2 \leq k \leq s-1$*.*

Proof. In view of Corollary 1.5, it is evident that the ssi-graph of those graphs containing C_k where $k \geq 2s+2$ is not chordal. Let $v_1, v_2, \dots, v_k$ where $1 \leq k \leq s-1$ be the vertices of the path P_k of the tadpole graph $T_{2s+2-k,k}$ and let u_i where $1 \leq i \leq 2s+2-k$ denote the vertices of the cycle C_{2s+2-k}. Suppose $u_1 \sim v_k$. Then the s-arcs $v_1v_2\dots v_ku_1u_2\dots u_{s+1-k}$, $v_1v_2\dots v_ku_1u_{2s+2-k}\dots u_{s+2}$, $v_2v_3\dots v_ku_1u_2\dots u_{s+2-k}$ and $v_2v_3\dots v_ku_1u_{2s+2-k}\dots u_{s+1}$ of G induce a C_4 in $A_s(G)$ since $1 \leq k \leq s-1$.

For the twin-tailed tadpole graphs, let $u_i; 1 \leq i \leq 2s+2-k$ (resp. $u_i; 1 \leq i \leq 2s+3-k$) denote the vertices of the cycle C_{2s+2-k} (resp. C_{2s+3-k}), $v_1, v_2, \dots, v_k$ and $w_1, w_2, \dots, w_k$ where $1 \leq k \leq s-1$ (resp. $2 \leq k \leq s-1$), be the vertices of the two k-paths attached to C_{2s+2-k} (resp. C_{2s+3-k}). The vertex sets that induces a 4-cycle in $A_s(G)$ are $\{v_1v_2\dots v_ku_1u_2\dots u_{s+1-k}$, $w_1w_2\dots w_ku_{s-k+2}u_{s-k+1}\dots u_2$, $w_1w_2\dots w_ku_{s-k+2}u_{s-k+3}\dots u_{2s+2-2k}$, $v_1v_2\dots v_ku_1u_{2s+2-2k}\dots u_{s+3-k}\}$ and $\{v_1v_2\dots v_ku_1u_2\dots u_{s+1-k}$, $w_1w_2\dots w_ku_{s-k+2}u_{s-k+1}\dots u_2$, $w_1w_2\dots w_ku_{s-k+2}u_{s-k+3}\dots u_{2s+2-2k}$, $v_1v_2\dots v_ku_1u_{2s+3-2k}\dots u_{s+4-k}\}$ respectively. $\square$

Theorem 1.5. *The* 1*si-graph of a graph* G *is chordal if and only if* G *is a tree or the circumference of* G *is* 3*.*

Proof. If G is not a tree and G contains a k-cycle where $k > 3$, it follows that the edges of the k-cycle which are vertices of $A_1(G)$ induce a k-cycle in $A_1(G)$. Hence, if $A_1(G)$ is chordal, G is either a tree or the circumference of G is 3. An immediate consequence of

Corollary 1.4 and Lemma 1.2 implies that the largest induced cycle in the ssi-graph of trees or graphs with circumference 3 is C_3. Moreover, on further analysis of Lemma 1.2, graphs satisfying the first and second condition for which the ssi-graphs exists are $K_{1,n}$, C_3 and P_3. □

Theorem 1.6. *The 2si-graph of any graph G is chordal if and only if G does not contain the following subgraphs:*

(i) cycle $C_k; k \geq 6$,
(ii) tadpole graph $T(5,1)$, and
(iii) twin-tailed tadpole graph $T_2(4,1)$.

Proof. By Lemma 1.3, if $A_2(G)$ is chordal, then G does not contain any of C_k with $k \geq 6$, $T_{5,1}$ and $T_2(4,1)$. It is essential that we prove that these are the only subgraphs that must not exist in G so that the 2si-graph is chordal. Since $A_2(G)$ is chordal, $A_2(G)$ is either a tree or the largest induced cycle contained in $A_2(G)$ is C_3. Suppose the circumference of G is either 4 or 5. If the circumference of G is 4, then by Corollary 1.3, $A_2(G)$ is C_5-free. One can observe that if G contains a C_4, $T_{4,1}$ or a C_4 with two adjacent vertices having branches, but not $T_2(4,1)$, then a C_4 is not induced in $A_2(G)$. Similarly if G contains a C_5, but not $T_{5,1}$, then clearly, the order of G is at most 5 implying that neither a C_4 nor a C_5 is induced in $A_2(G)$.

Conversely, suppose G is a graph that does not have the given subgraphs. This implies that the circumference of G is at most 5 or G is a tree. Suppose G is a graph with circumference at most 5 and G does not contain both $T_{5,1}$ and $T_2(4,1)$. Then, G can have a C_4 or C_5 with no branches at any vertex, $T_{4,1}$ or a 4-cycle with branches at exactly two adjacent vertices of the 4-cycle. The maximum order of an induced cycle in $A_2(G)$ for each of these cases is a C_3. Similarly, if the circumference is 4 and if G does not have $T_2(4,1)$, then clearly $A_2(G)$ is chordal. □

2. ALGORITHM TO FIND THE VERTICES OF THE SSI-GRAPH OF A GRAPH

The problem to compute the order of the ssi-graph of a graph remains open for $s > 2$. However, an algorithm to find all the vertices of the ssi-graph of a graph, for possible values of s, is presented below. Based on the non-empty intersection between any two s-arcs of a graph G, which are vertices of the ssi-graph of G, the ssi-graph of G can be obtained. Identify the detour diameter of the graph and label it as s^*. The possible values of s are $1, 2, \ldots, s^*-1$. Let A be the set of all vertices of G with detour eccentricity greater than s.

Step 1: Select an arbitrary vertex, say u_1, of G with detour eccentricity greater than s and let $u_2 \in N(u_1)$ with $\deg_G(u_2) > 1$, be the second vertex of the s-arc.

Step 2: For $2 \leq i \leq s$, select any neighbour of u_i, say $u_{i+1} \in N(u_i) - \{u_{i-1}, u_{i-2}, \ldots, u_1\}$, with $\deg_G(u_{i+1}) > 1$. Continuing this process, the $(s+1)$-th vertex of this s-arc is u_{s+1}. If $N(u_{s+1}) - \{u_1, u_2, \ldots, u_s\} \neq \emptyset$, label this s-arc as α_1; else, move to Step 1 with the next vertex of A.

Step 3: Let $\alpha_1, \alpha_2, \ldots, \alpha_p$ be the vertices obtained by considering the sequences $\{u_1, u_2, \ldots u_s\}, \{u_1, u_2, \ldots u_{s-1}\}, \{u_1, u_2, \ldots u_{s-2}\}, \ldots, \{u_1, u_2, \ldots, u_{s-i+2}\}$. Note that the number of vertices of $A_s(G)$ generated by each sequence may vary according to the structure of the graph. Now, proceed to Step 3(a).

(a) Consider the sequence $\{u_1, u_2, \ldots, u_{s-i+1}\}$. Let $u_{ji} \in N(u_{s-i+1}) - \{u_1, u_2, \ldots, u_{s-i}, u_{s-i+2}\}$ with $\deg_G(u_{ji}) > 1$. If such a u_{ji} exists, follow the next sub-step. If no such u_{ji} exists, repeat Step 3(a) with the next sequence.

(b) For the sequence $\{u_1, u_2, \dots, u_{s-i+1}, u_{ji}\}$, let $u_{j(i+1)} \in N(u_{ji}) - \{u_1, u_2, \dots, u_{s-i}, u_{s-i+1}\}$ with $\deg_G(u_{j(i+1)}) > 1$. If no such $u_{j(i+1)}$ exists, repeat Step 3(b) with the other possible neighbours u_{ji} of u_{s-i+2}. If such $u_{j(i+1)}$'s exist, proceed to Step 3(c). Once all neighbours $u_{j(i+1)}$ of u_{ji} are exhausted, move to Step 3(a) and continue the process with the next possible neighbour u_{ji} of u_{s-i+1}.

(c) Consider the sequence $\{u_1, u_2, \dots, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{jk}\}$, where $i \leq k \leq s-1$. Let $u_{j(k+1)} \in N(u_{jk}) - \{u_1, u_2, \dots, u_{s-i}, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{j(k-1)}\}$ with $\deg_G(u_{j(k+1)}) > 1$. If such a $u_{j(k+1)}$ exists, proceed with the next iteration; else, repeat Step 3(c) to find the other possible neighbours of u_{jk} and re-label them as $u_{j(k+1)}$. Once all neighbours $u_{j(k+1)}$ of u_{jk} are exhausted, move to the previous iteration and continue the process with the next possible neighbour u_{jk} of $u_{j(k-1)}$. Once this sequence $\{u_1, u_2, \dots, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{js}\}$ on s vertices is obtained, proceed to Step 3(d).

(d) For the sequence $\{u_1, u_2, u_3, \dots, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{js}\}$, let $u_{j(s+1)} \in N(u_{js}) - \{u_1, u_2, u_3, \dots, u_{s-i}, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{j(s-1)}\}$ with $\deg_G(u_{j(s+1)}) > 1$ and $N(u_{j(s+1)}) - \{u_1, u_2, \dots, u_{s-i}, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{j(s-1)}, u_{js}\} \neq \emptyset$. If such a $u_{j(s+1)}$ exists, label the s-arc as α_{p+1}. Repeat this procedure till all such $u_{j(s+1)}$'s are exhausted. After finding vertices $\alpha_{p+2}, \dots, \alpha_t$, move to Step 3(d) and continue with the next possible neighbour u_{js} of $u_{j(s-1)}$. If no such $u_{j(s+1)}$ exists, proceed with the next iteration considering the next possible neighbour u_{js} of $u_{j(s-1)}$.

Step 4: Once the process is completed for all neighbours of u_1, repeat Step 1 with the next vertex in A. Terminate the process once all vertices in A are exhausted.

Repeat all steps with the next vertex in A and continue the process until we get all possible vertices of $A_s(G)$.

Theorem 2.7. *The set S of s-arcs of G obtained by the above algorithm is the vertex set of the ssi-graph of G.*

Proof. Note that since the detour eccentricity of u_i is greater than s, there exists s-arcs on distinct vertices with the first vertex as u_i that can be shunted onto some other s-arc on distinct vertices of G. The s-arc α_1 obtained from the above algorithm is a sequence of $s+1$ vertices of G that are distinct. Since $N(u_{s+1}) - \{u_1, u_2, \dots, u_s\} \neq \emptyset$, α_1 can be shunted onto another s-arc on distinct vertices of G. Hence, α_1 is a vertex of $A_s(G)$. The sequences obtained from $\{u_1, u_2, \dots, u_{s-i+1}\}$ at each iterative step have distinct vertices. After each iterative step, since $N(u_{j(s+1)}) - \{u_1, u_2, \dots, u_{s-i}, u_{s-i+1}, u_{ji}, u_{j(i+1)}, \dots, u_{j(s-1)}, u_{js}\} \neq \emptyset$ and the vertices in the sequence are distinct, this implies that the sequence of vertices obtained is an s-arc of G on distinct vertices, that can be shunted onto another s-arc of G on distinct vertices. Hence, this is a vertex of $A_s(G)$. If $u_i u_{i+1} \dots u_{i+s-1} u_{i+s}$ is a vertex obtained by using the algorithm with the first vertex as u_i, then the vertex $u_{i+s} u_{i+s-1} \dots u_{i+1} u_i$, if exists in $V(A_s(G))$, is obtained by using the algorithm with the first vertex of the s-arc as u_{i+s}. Hence, using the algorithm for every vertex of G with detour eccentricity greater than s guarantees that all vertices of $A_s(G)$ are obtained without any vertex being repeated. □

The detour diameter of the graph G in Figure 2 is 7, since G has 8 vertices and contains a Hamiltonian path $u_8u_7u_1u_6u_5u_4u_3u_2$. Note that the diameter of G is All vertices of G have detour eccentricity greater than 3. For the vertex u_7 of G, we consider the neighbour u_1 of u_7 which is not a pendant vertex of G. Let the first vertex of $A_3(G)$ be $u_7u_1u_5u_4$. As per the algorithm, we consider the sequence $u_7u_1u_5$ and a vertex in $N(u_5) - \{u_7, u_1, u_4\}$. The vertices in $N(u_5) - \{u_7, u_1, u_4\}$ are u_6 and u_3. Since $N(u_6) - \{u_7, u_1, u_5\} = \emptyset$ and $N(u_3) - \{u_7, u_1, u_5\} \neq \emptyset$, the next vertex of $A_3(G)$ is $u_7u_1u_5u_3$. Since all possible neighbours of

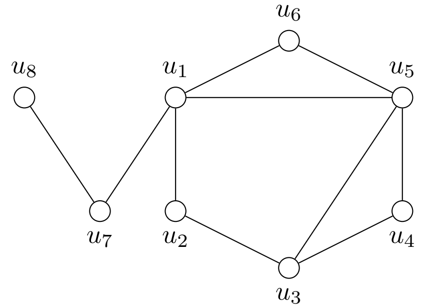


FIGURE 2. A graph G illustrating the algorithm.

u_5 are exhausted, we now consider the sequence u_7u_1 and the internal vertices $u_2, u_6 \in N(u_1) - \{u_7, u_5\}$. The next sequence to be considered is $u_7u_1u_2$. Since $u_3 \in N(u_2) - \{u_7, u_1\}$ and $N(u_3) - \{u_7, u_1, u_2\} \neq \emptyset$, the next vertex of $A_3(G)$ is $u_7u_1u_2u_3$. Since all possible neighbours of u_2 are exhausted, we consider the sequence $u_7u_1u_6$. Repeating the same procedure as above, the next vertex of A_3 of G is $u_7u_1u_6u_5$. All possible neighbours of u_6 have been exhausted. It can be observed that all possible neighbours of u_1 have been exhausted as well. Hence, we continue the process by considering those neighbours of u_7 which are not pendant vertices of G. In this case, we have obtained all vertices of $A_3(G)$ where the first vertex is u_7. To obtain the other vertices of $A_3(G)$, the algorithm is used with other vertices of G.

Based on the algorithm discussed above, the 45 vertices of the graph G in Figure 2 are $u_1u_6u_5u_3$, $u_1u_6u_5u_4$, $u_1u_5u_4u_3$, $u_1u_2u_3u_4$, $u_1u_2u_3u_5$, $u_2u_1u_6u_5$, $u_2u_1u_5u_4$, $u_2u_1u_5u_3$, $u_2u_3u_4u_5$, $u_2u_3u_5u_1$, $u_2u_3u_5u_6$, $u_3u_2u_1u_6$, $u_3u_2u_1u_7$, $u_3u_2u_1u_5$, $u_3u_5u_1u_7$, $u_3u_4u_5u_6$, $u_3u_4u_5u_1$, $u_3u_5u_6u_1$, $u_3u_5u_1u_2$, $u_4u_5u_1u_7$, $u_4u_5u_1u_2$, $u_4u_3u_2u_1$, $u_4u_3u_5u_1$, $u_4u_3u_5u_6$, $u_4u_5u_3u_2$, $u_4u_5u_6u_1$, $u_5u_6u_1u_7$, $u_5u_6u_1u_2$, $u_5u_1u_2u_3$, $u_5u_4u_3u_2$, $u_5u_3u_2u_1$, $u_6u_1u_5u_4$, $u_6u_1u_5u_3$, $u_6u_1u_2u_3$, $u_6u_5u_1u_7$, $u_6u_5u_1u_2$, $u_6u_5u_4u_3$, $u_6u_5u_3u_2$, $u_7u_1u_6u_5$, $u_7u_1u_5u_4$, $u_7u_1u_5u_3$, $u_7u_1u_2u_3$, $u_8u_7u_1u_6$, $u_8u_7u_1u_5$ and $u_8u_7u_1u_2$. Based on the non-empty intersection between any two of these 3-arcs of G, the 3si-graph of G is obtained.

3. CONCLUSIONS

The chordality of the ssi-graph of a graph G for any possible values of s was studied. Also, an algorithm to obtain all vertices of the ssi-graph of G was also developed. These findings contribute a clearer understanding on the structure of the ssi-graph of a graph and is an extension of the topic studied in [8]. Further, the results obtained from this study aid in exploring the perfection of the ssi-graph of a graph, which is currently under study.

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